



# Formative feedback on linear equations

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Literary review

Formative  
feedback

STACK

Linear  
equations

# Literary review: formative feedback



## Feedback

“Information [...] regarding aspects of one’s performance or understanding”



## Types

- ❑ Sumative/**formative**
- ❑ Conceptual/**procedural** /based on practices
- ❑ **Immediate**/delayed...



## Given...

...after initial instruction.  
“Feedback has no effect in a vacuum”.  
**It has to be useful.**

# Literary review: STACK



Answer: algebraic expressions



Encoding of typical mistakes



Specific feedback

$$\begin{aligned} & 3(-2x + 1) = -x \\ ? & -6x + 1 = -x \\ \Leftrightarrow & 1 = 5x \\ \Leftrightarrow & x = \frac{1}{5} \end{aligned}$$

Check whether you correctly applied the distributive property. Did you multiply the factor outside of the parentheses by each of the ones inside?



# Literary review: linear equations

**Distributive property**

Expand  $A(Bx+C)$

**Addition**

Add/subtract the same term on both sides

**Multiplication**

Multiply/divide by the same factor on both sides

**Common denominator**

Add fractional numbers

**Coeff-var separation**

$Ax \rightarrow A+x$

**Arithmetic**

Basic arithmetic operations

Pérez et al. (2019)



# Main goals

To what extent is formative feedback through STACK effective for students overcoming difficulties on linear equations?




# Methodology

## Sample

132 students:

- 39 in secondary school (aged 14-16)
- 93 who have finished secondary school

Average achievers – some especially struggled

 Cantabria, Spain



# Methodology

## Equations

a)  $3(-2x+1)=-x$

b)  $\frac{7}{2}x = 14$

c)  $-4x=16$

d)  $-2(3x-4)=10$

e)  $\frac{x}{3} + 2x = 7$

f)  $-3x+5=17$

Pérez et al. (2019)

## Feedback

$$3(-2x+1) = -x$$

?  $-6x+1 = -x$

$\Leftrightarrow 1 = 5x$

$\Leftrightarrow x = \frac{1}{5}$

Check whether you correctly applied the distributive property. Did you multiply the factor outside of the parentheses by each of the ones inside?

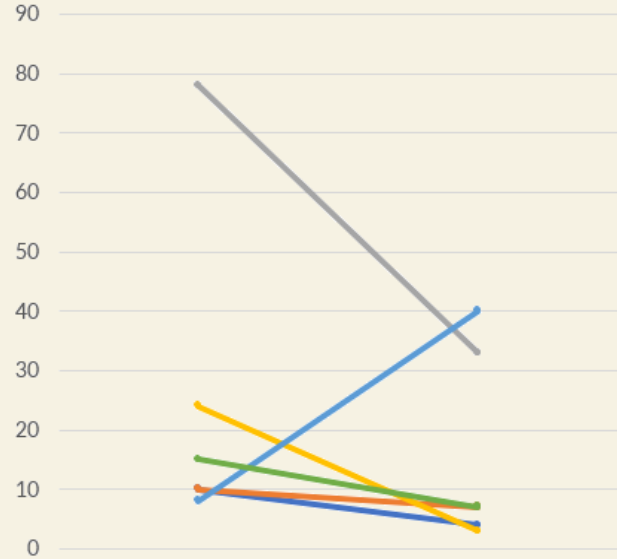


# Results

## Mistakes

- Distributive property ●
- Addition ●
- Multiplication ●
- Common denominator ●
- Coeff-var separation ●
- Arithmetic ●

Number of mistakes



Start

Finish

# Results

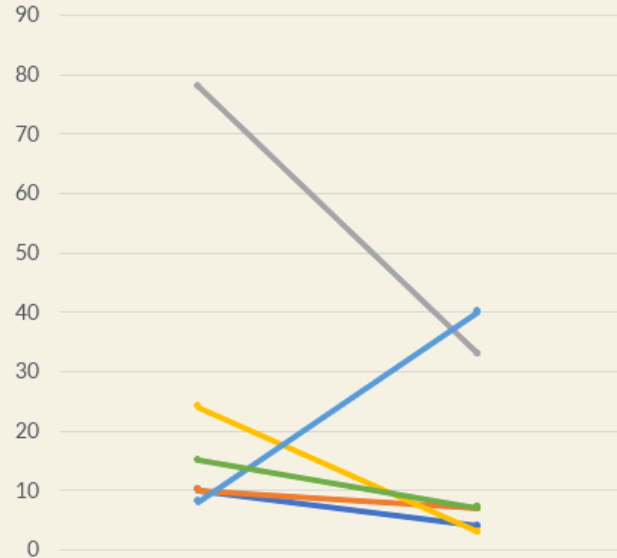
Distributive  
property  
Coeff-var  
separation

Older students who  
have forgotten

Addition  
Arithmetic

Distractions

Number of mistakes



Start

Finish

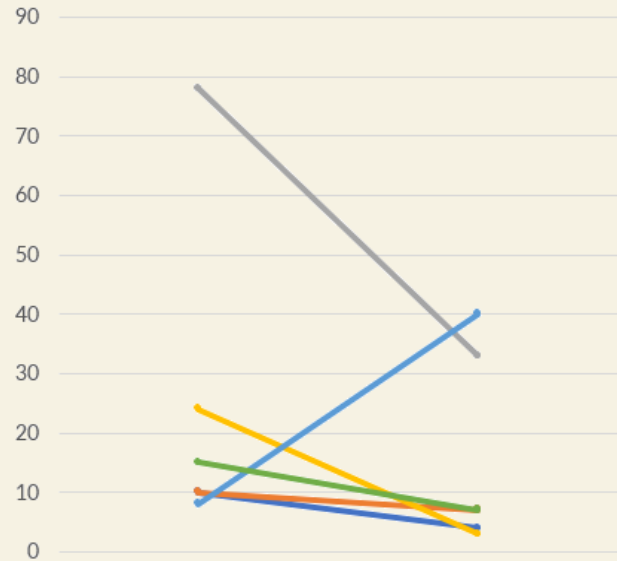
# Results

## Multiplication

Students tried to solve  $Ax=B$  as:

- $x=B/A$
- $x=-B/A$
- $x=A/B$
- $x=-A/B$

Number of mistakes



Start

Finish

# Results

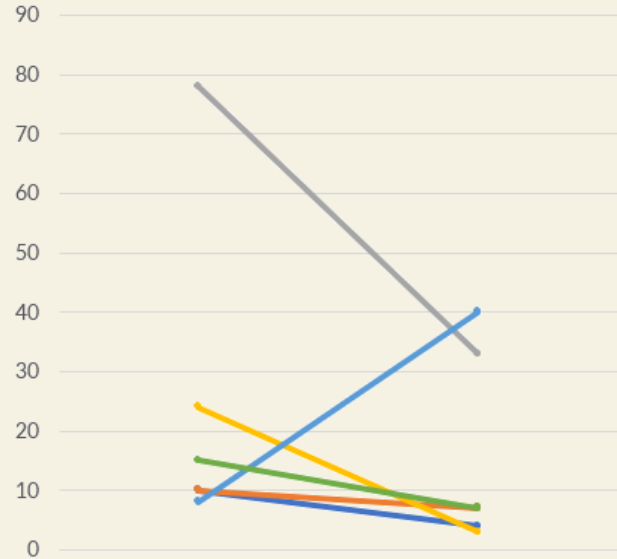
## Common denominator

Students avoided using common denominator:

$$\begin{aligned} \frac{7}{2}x &= 14 \\ \Leftrightarrow 7x &= 28 \\ \Leftrightarrow x &= \frac{28}{7} \\ \Leftrightarrow x &= 4 \end{aligned}$$

$$\begin{aligned} \frac{x}{3} + 2x &= 7 \\ \Leftrightarrow \frac{x}{3} &= 7 - 2x \\ \Leftrightarrow x &= 3(7 - 2x) \\ \Leftrightarrow x &= 21 - 6x \\ \Leftrightarrow 7x &= 21 \\ \Leftrightarrow x &= 3 \end{aligned}$$

Number of mistakes



Start

Finish



# Conclusion

To what extent is formative feedback through STACK effective for students overcoming difficulties on linear equations?



# Conclusion

## Mistakes

Fewer and easier to correct

# Why?



# Conclusion

## Mistakes

Fewer and easier to correct

### Feedback

- Immediate
- Only after the mistakes
  - Focused on the task
- Short, clear and specific

Computer → objective

Try again

Not providing grades

Prior  
knowledge

STACK

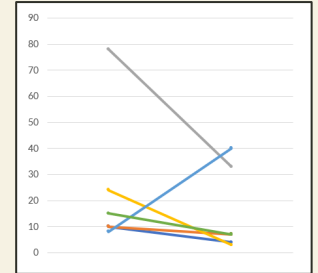
# Conclusion

## Mistakes

Fewer and easier to correct

## Further studies

Different sample/feedback







## References

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