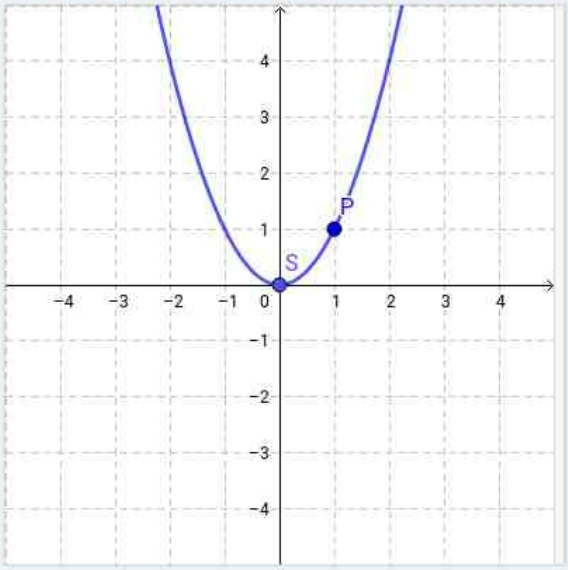
# Merging GeoGebra and STACK : Technical advancement and didactic reflection

Guido Pinkernell ∘ Pädagogische Hochschule Heidelberg
STACK Community Conference ∘ April 2023

Move the points S und P,
such that the graph fits with
$f(x) = -3 \cdot (x+1)^2 - 2$.




Check

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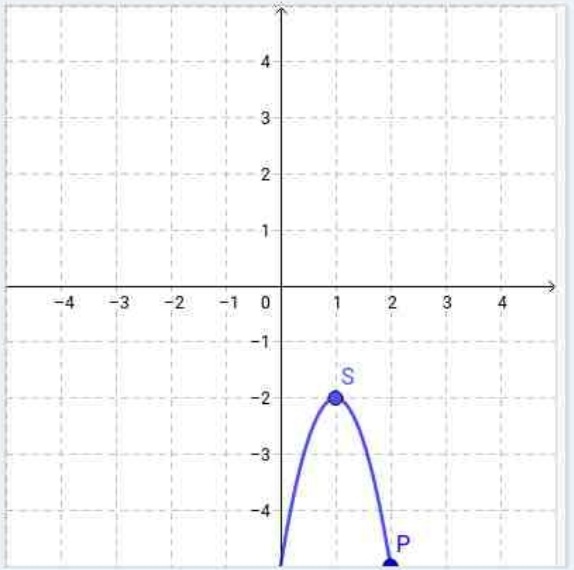
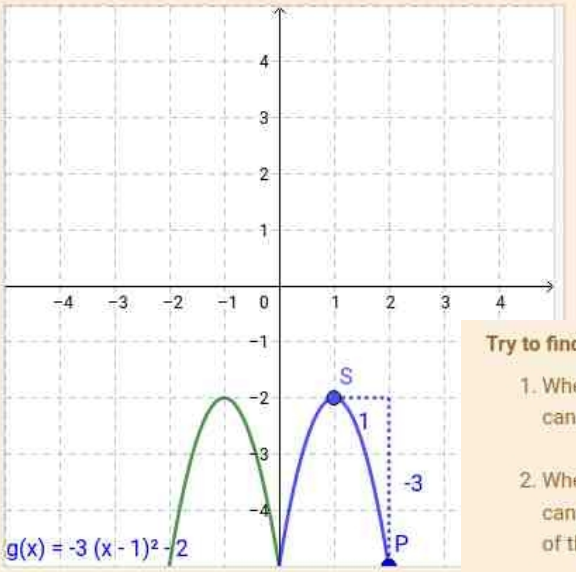



Check

**Wrong, too bad!**

The green graph would be correct.




**Why?**

You can find out yourself.

Correct your blue graph
and watch how the expression changes.

**Try to find answers to the following questions:**

1. Where in the expression
   can you see the coordinates of the vertex?

2. Where in the expression
   can you see a value for the opening
   of the parabola?

**Do you have an idea already?**

Then try the task again.

**Or wait 30 seconds**
**after which a full solution will appear:**

Musterlösung

STACK

- randomised questions
- differentiated feedback based on automatic answer analysis
- highly adaptable (for experienced in coding)
- large academic community

GeoGebra

- multimodal, dynamic, and interactive information
- pad for creating (defining, sketching,…) mathematical objects
- highly accessible (for noncoders)
- large school community

Pinkernell, G., Diego Mantecón, J. M., Lavicza, Z., Sangwin, C. (2023): AuthOMath: Combining the strengths of STACK and GeoGebra

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# overview

1. introductory review
2. AuthOMath
    - partners and places
    - objectives
3. didactic concept




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AuthOMath (2022-2024)

Authoring Online Material with Multimodal, Dynamic and Interactive Applets and Automated Feedback for Learning Math



www.authomath.org
mathebruecke.pinkernell.online

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University of Education Heidelberg : Guido Pinkernell
University of Cantabria Santander : Jose Manuel Diego Mantecon
University of Edinburgh : Chris Sangwin
Johann-Kepler-Universität Linz : Zsolt Lavica
Geogebra GmbH (associated)

for more details about
partners, persons, and places
see www.authomath.org



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# 2. AuthOMath

AuTo

- a moodle based authoring tool
  for randomized interactive and dynamic
  multimodal mathematical tasks
  with automatic adaptive feedback

DiCo

- a didactical concept
  for designing online based
  interactive learning material
  for use in mathematics teacher
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  which basically means to

  extend STACK to facilitate
  the implementation of
  GeoGebra applets into
  task and feedback



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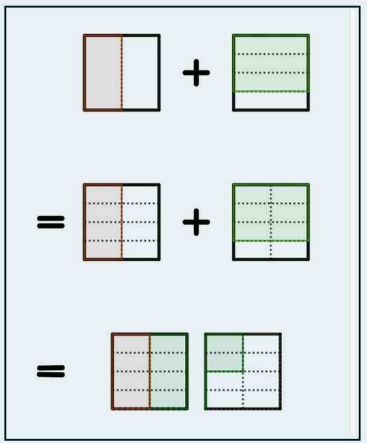
This visualization shows
how two fractions are added.

Translate into maths:



```
n1:rand([2,3,4,5]);
n2:rand_with_prohib(2,5,[n1]);
z1:rand(n1-1)+1;
z2:rand(n2-1)+1;
```

```
1  <br>This visualization shows<br>how two fractions are added.<br><br>Tra
2
3  <table style="border-collapse: collapse; width: 400px;" border="0">
4      <tbody>
5          <tr>
6              <td style="width: 180px; vertical-align: bottom; border-sty
7
8                  [[geogebra set="n1,n2,z1,z2,x1,y1,x2,y2"]]
9                  params["material_id"] = "yqhjpr2c";
10                 params["width"] = 450;
11                 params["height"] = 550;
12                 params["borderColor"] = "rgba(0, 0, 0, 0)";
13                 params["transparentGraphics"]= true;
14                 params["scale"] = 0.5;
15                 [[/geogebra]]
16
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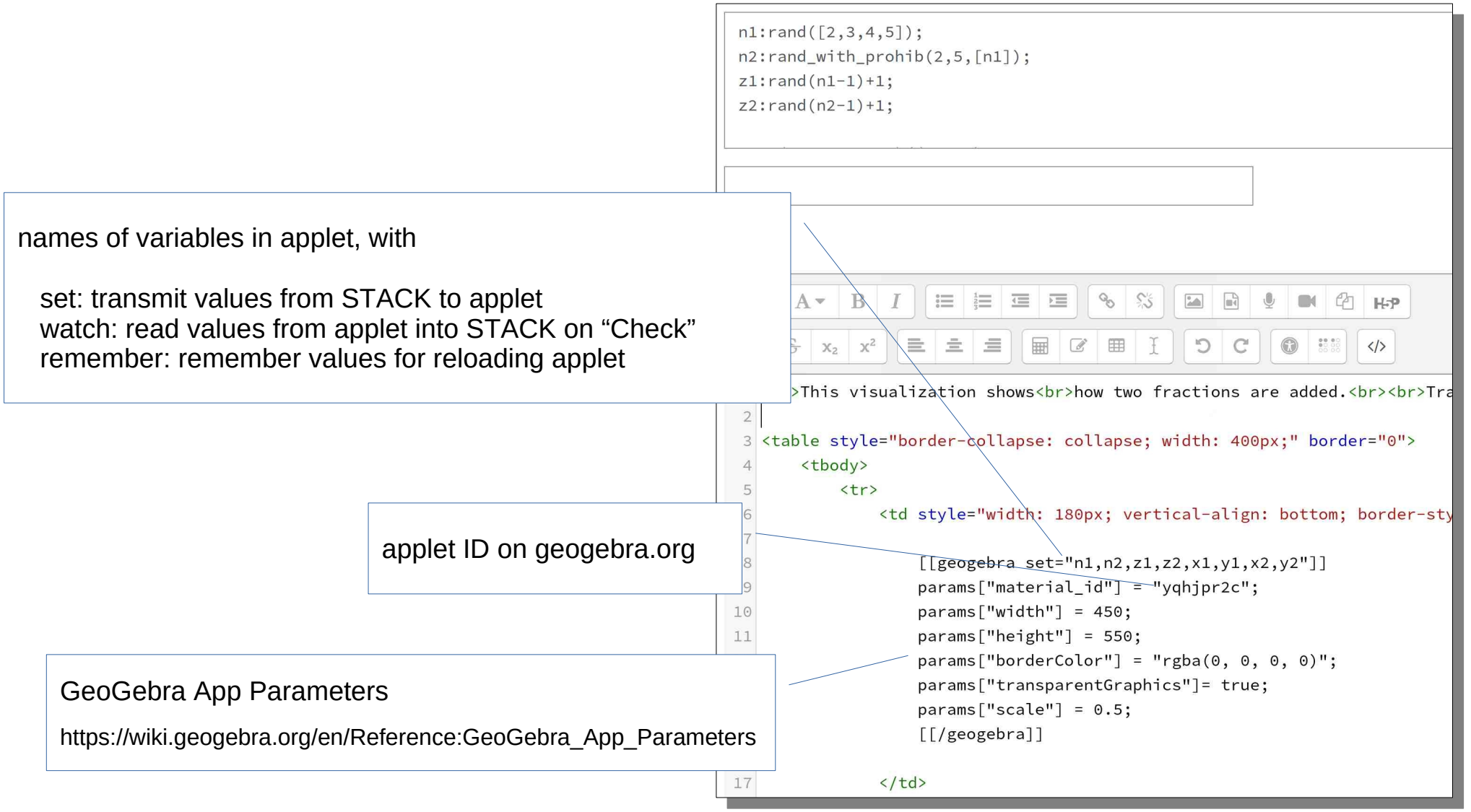
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names of variables in applet, with
set: transmit values from STACK to applet
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applet ID on geogebra.org
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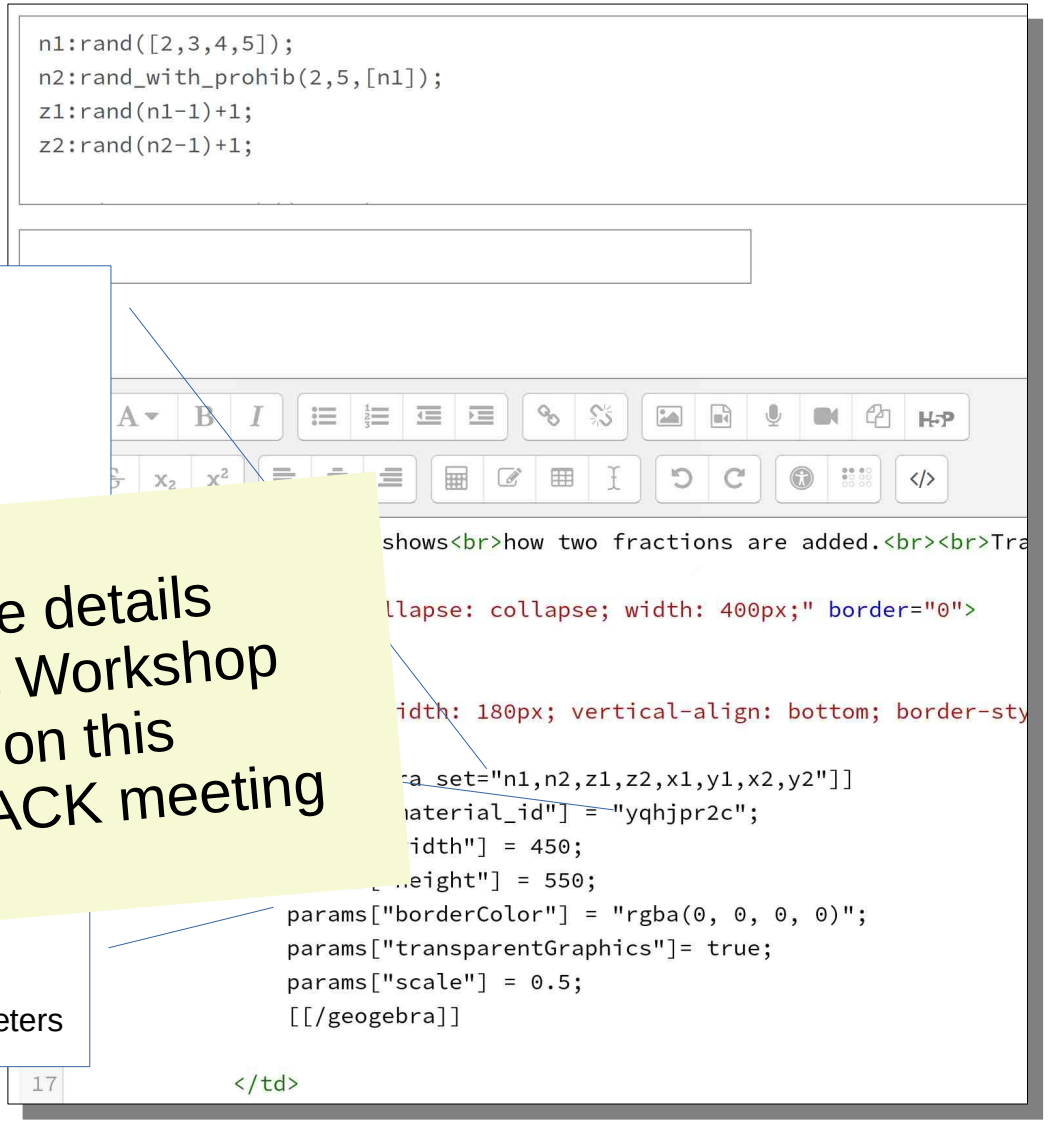


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applet

for more details
see Tim’s Workshop
here on this
Tallinn STACK meeting

GeoGebra App Parameters

https://wiki.geogebra.org/en/Reference:GeoGebra_App_Parameters

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# 3. didactic concept : work in pro

- takes up the
  specific affordances of
  GeoGebra and STACK

  to initiate didactic reflection
  on the digitalisation of
  mathematical tasks

  in teacher education

"You want to use
GeoGebra and STACK
for creating
multimodal and interactive
math tasks
with adaptive feedback.

What is it
that you need to think about
before you do the programming
(if this is you...)"



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# focussing variance in mathematical thinking

- ...in question design, enabled by randomisation
- ...of feedback structure, enabled by adaptive answer analysis

⇒ didactic a priori analysis of the potential range of...

- solving strategies for word problems
- example spaces for tasks asking for giving examples of math concepts
- misconceptions and errors for basic math tasks

Ríos-San-Nicolás, Á., Diego-Mantecón, J. M., Ortiz-Laso, Z., & Pinkernell, G. (2022). The use of STACK for developing problemposing and -solving skills of future mathematics teachers.

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randomisation initiates need to reflect on

- mathematical coherence of randomization
- contextual coherence of randomization

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answer analysis for feedback initiates need to reflect on

- range of possible solution strategies
- hierarchy of strategies

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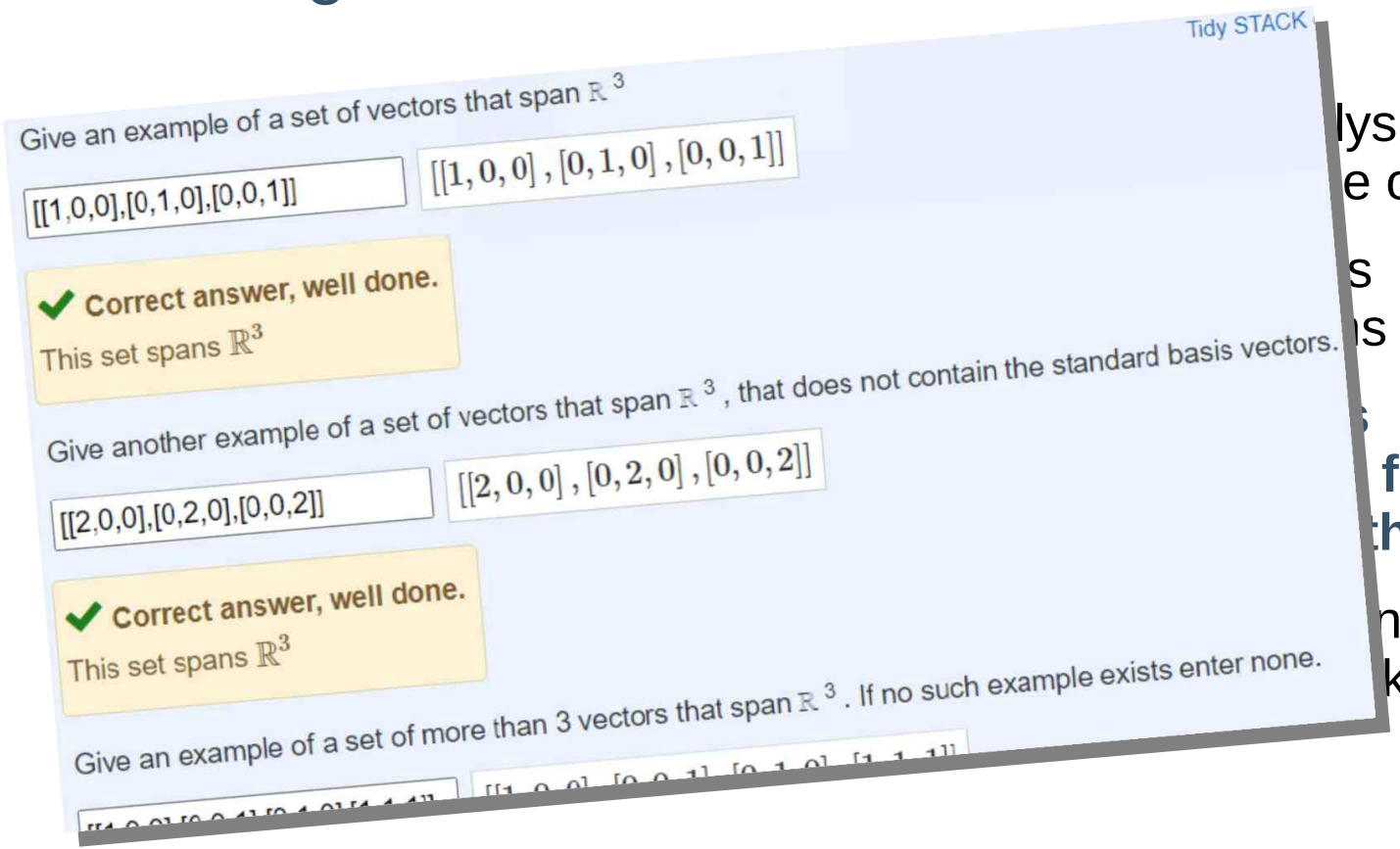


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# focussing activity and flexibility in mathematical thinking

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# focussing activity and flexibility in mathematical thinking

- ...in question and feedback design enabled by interactive applets

⇒ didactic a priori analysis of media use for fostering mathematical understanding

- representational flexibility for accessing a mathematical concept, which is abstract by nature
- interactivity for a manipulative access to the mathematical concept
- dynamic representation for accessing a mathematical concept as a class of examples

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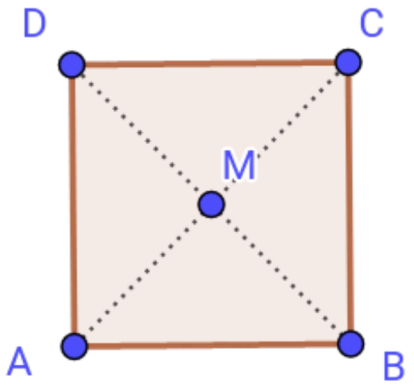


This is not a square.

Move points
to explore the range of appearances,
and then decide on what this quadrangle really is.

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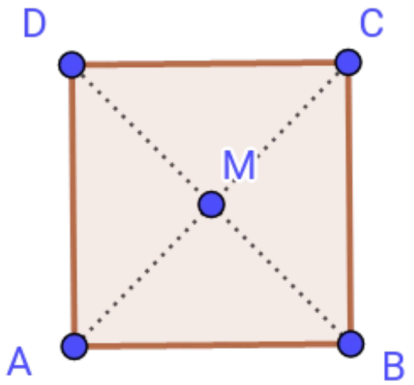


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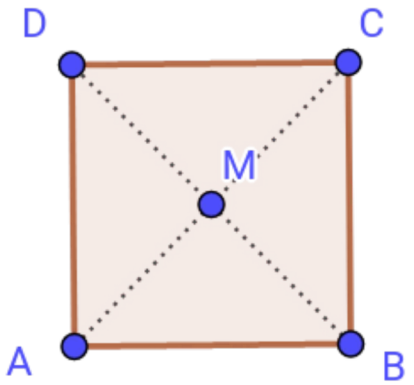


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# DiCo : organisational frame

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1. how to address the object of learning
2. how to address the subjects of learning
3. how to use the available media

# DiCo : organisational frame

1. **how to address the object of learning**
2. how to address the subjects of learning
3. how to use the available media

- mathematical definitions
- explanatory models which are both didactically and mathematically sound (“Grundvorstellungen”)
- range of desirable examples or solving strategies
- ...

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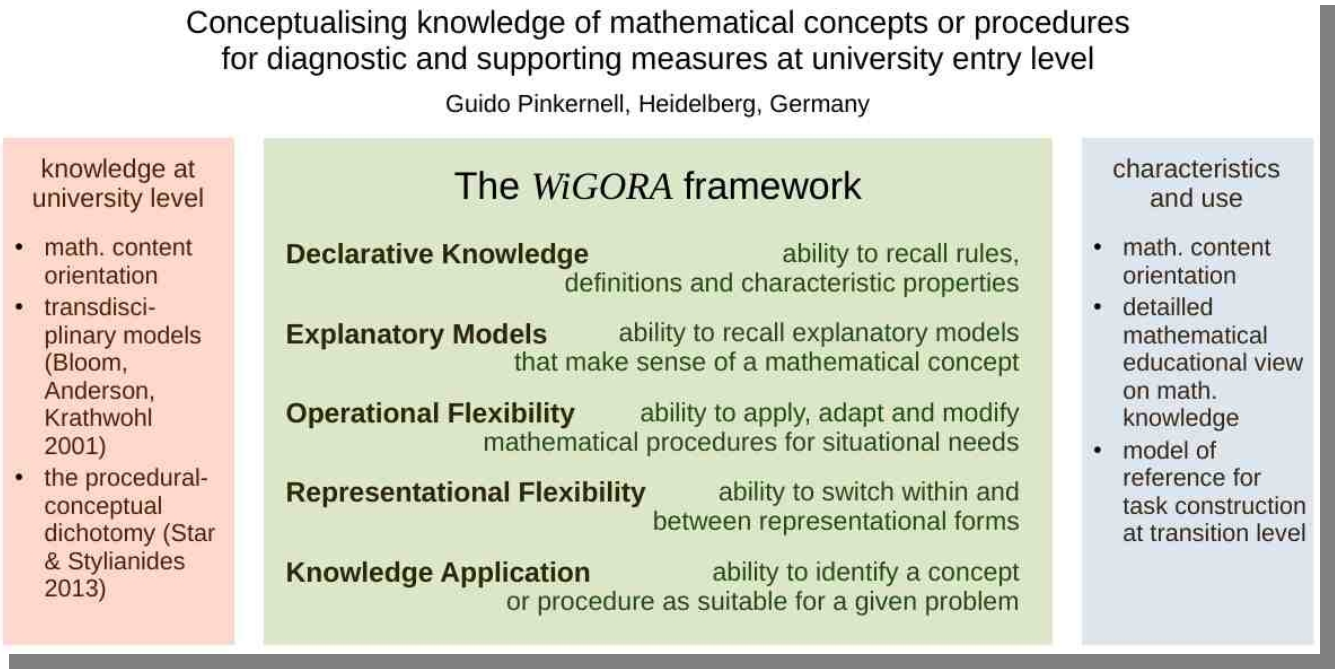


Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures

# DiCo : organisational frame

1. how to address the object of learning
2. **how to address the subjects of learning**
3. how to use the available media

- forms of understanding
  (range of strategies, example space, ...)
- typical misconceptions and errors
- suitable feedback, e.g.
  - **informing** the learner about how to work out the answer along a given procedure
  - **activating** the learner to work out the basis of understanding by her/himself

also:
“activating the learner to have her/him actually working with the feedback…”
(cp. discussion of Juma, Jones et al. yesterday)

# DiCo : organisational frame

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3. **how to use the available media**

the actual content, structure and design of

- textual information
- pictorial information

in question and feedback

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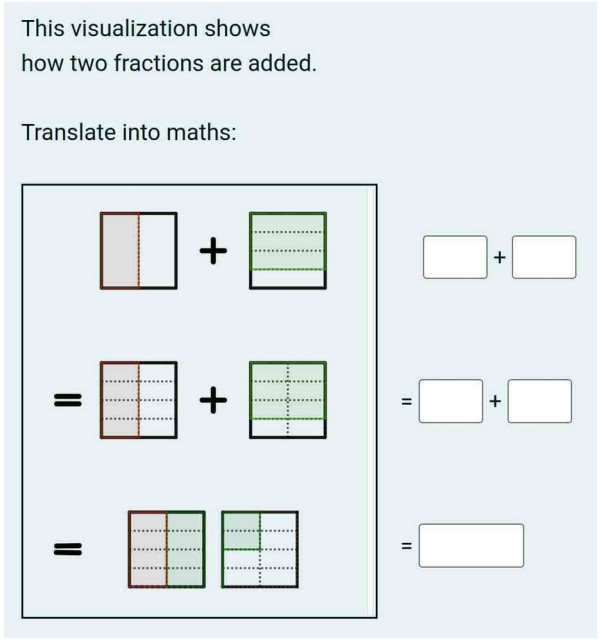


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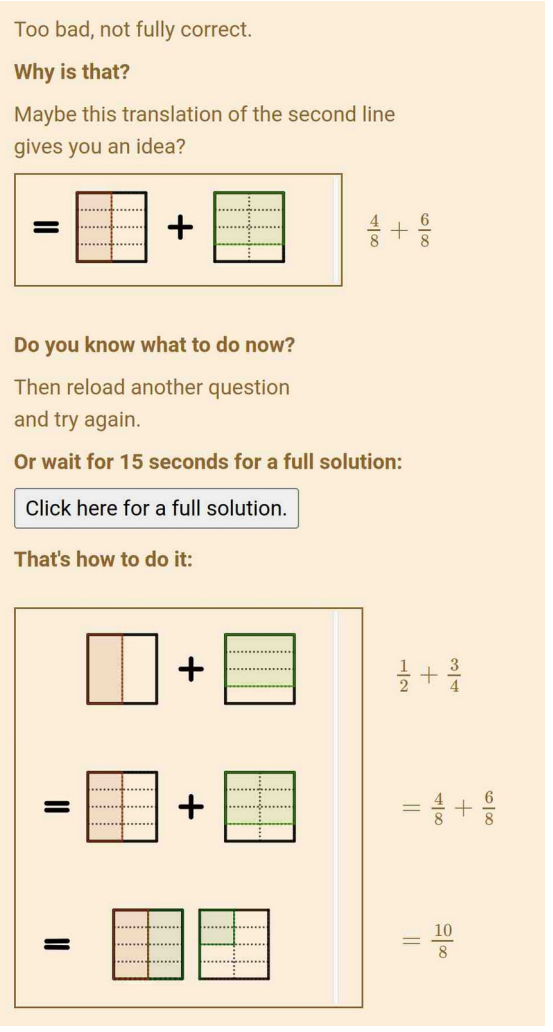

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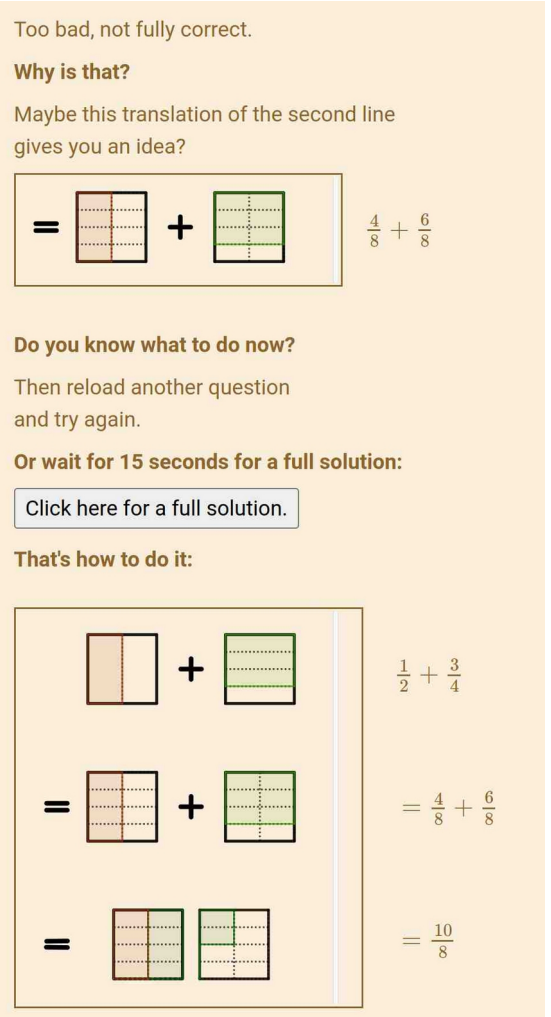


Pinkernell (2021): Aspects of Feedback Design ∘ Narciss (2008): Feedback Strategies for Interactive Learning Tasks

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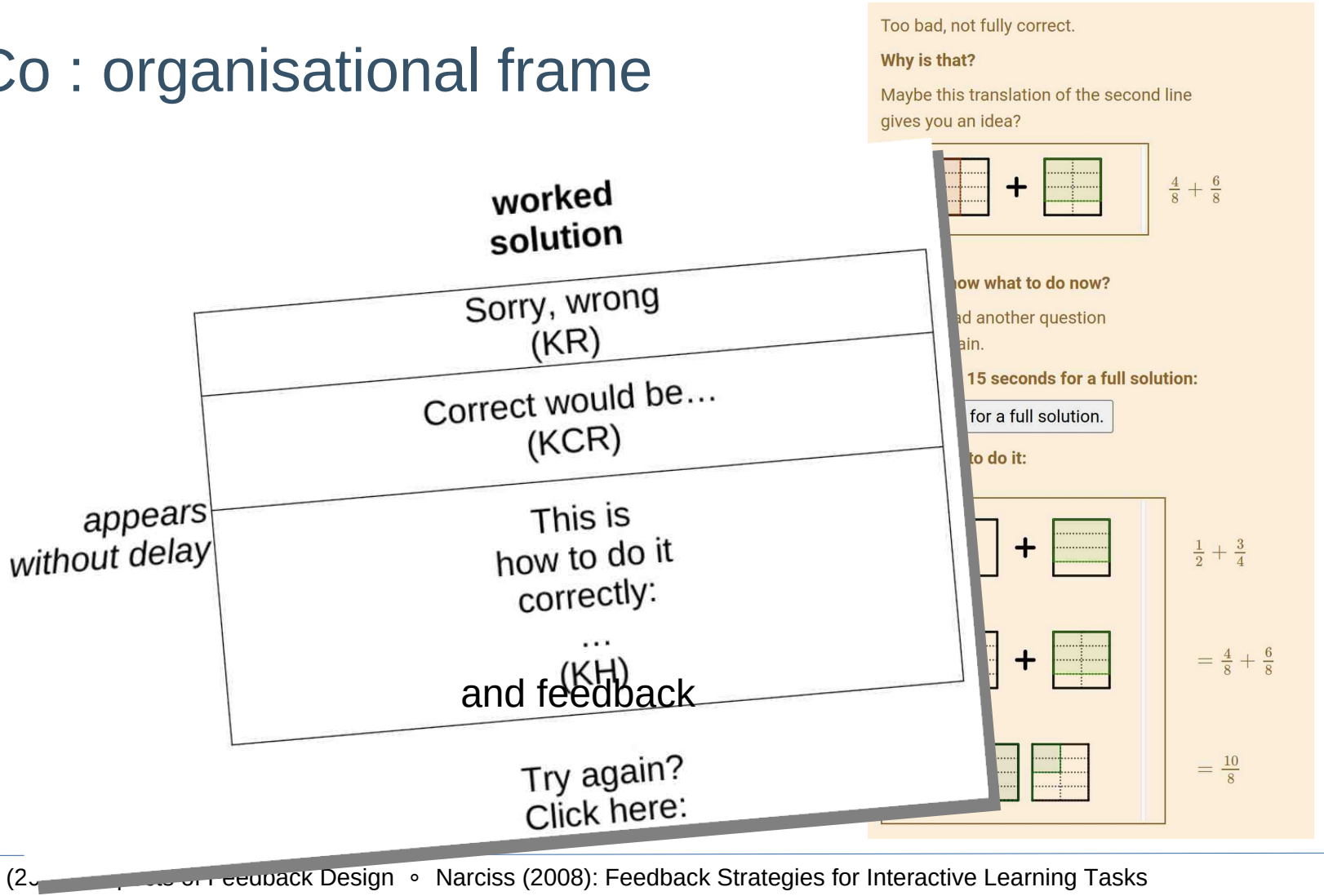

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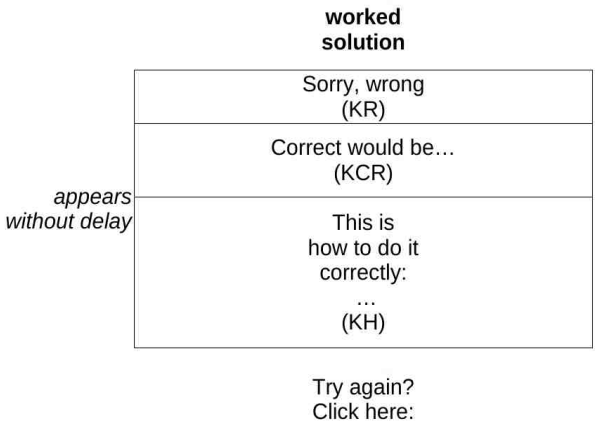


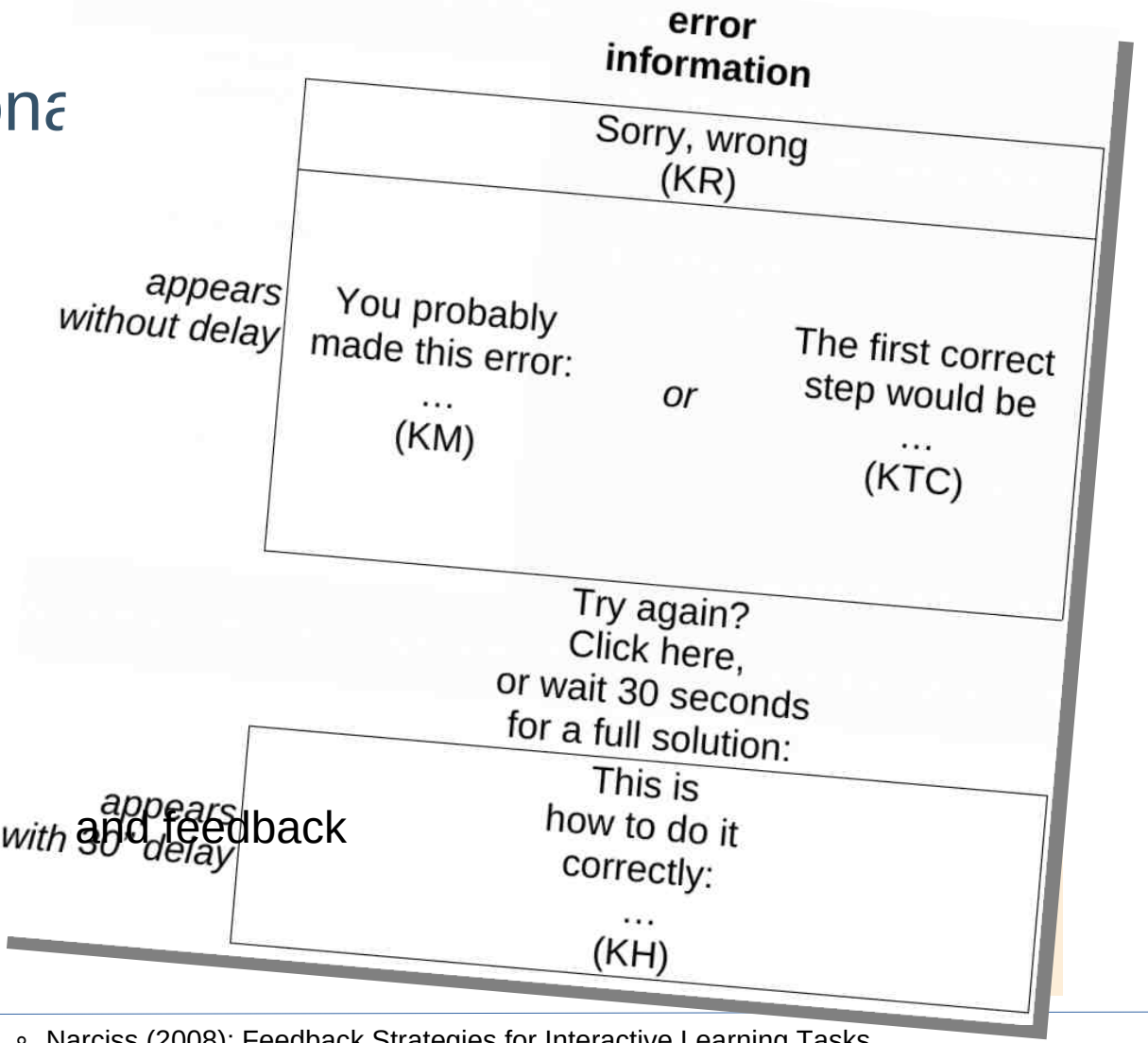


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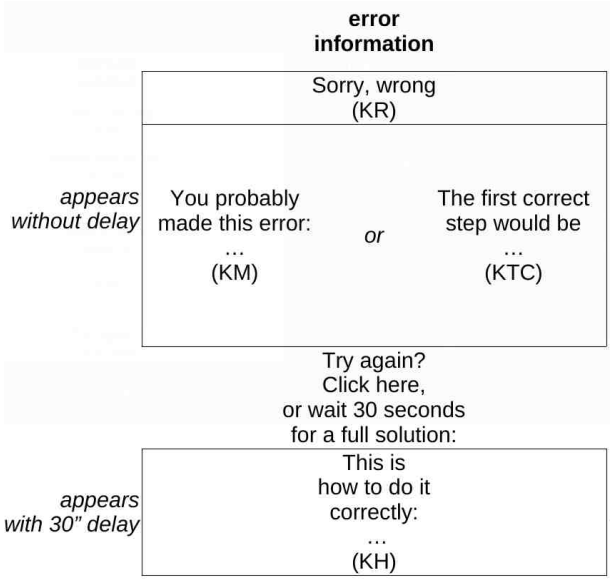


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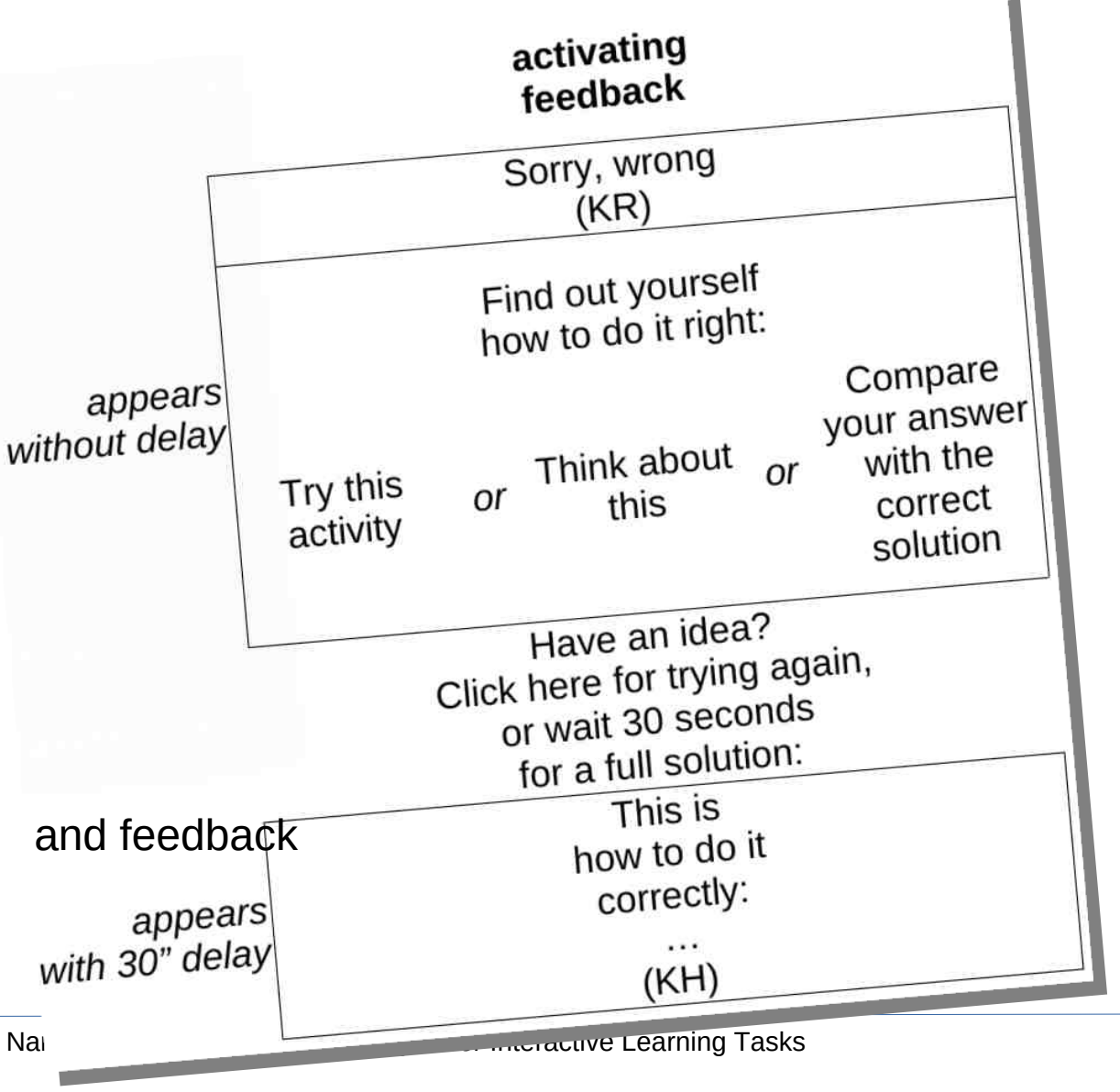

# summary

1. introductory review
2. AuthOMath
   - partners and places
   - objectives
3. didactic concept

ad 1.: “merging” means that
by extending STACK code to facilitate implementation of GeoGebra applets in STACK questions,
the specific strengths of both can come together in one authoring platform

ad 2.: two objectives, which are technical (code)
and didactic (didactic concept)

ad 3.: the didactic concept takes up
the specific affordances of STACK and GeoGebra for initiating didactic reflection in teacher education,

hence focussing students’ didactic reflection on

the variance in forms of
conceptualisations of mathematical concepts
or strategies of solving given problems,

activity and flexibility for
building a better understanding
of math concepts



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2. AuthOMath

Thank you
for your
kind attention!

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