Activating Feedback in Formative Assessment : From Receptive to Active Learning with Automated Feedback

Guido Pinkernell ∘ Pädagogische Hochschule Heidelberg
1st Northern e-Assessment Meeting 2023 Trondheim ∘ June 2023

overview

1. examples
2. theory
3. suggestions

The creation of these resources has been (partially) funded by the ERASMUS+ grant program of the European Union under grant No. 2021-1-DE01-KA220-HED-000032031. Neither the European Commission nor the project's national funding agency DAAD are responsible for the content or liable for any losses or damage resulting of the use of these resources.

examples

Give a quadratic expression
which has exactly the two roots -3 und -1 .

$f(x) =$ (x-3)*(x-1)

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer again...

examples

$\frac{1}{4} = \frac{5}{20}$

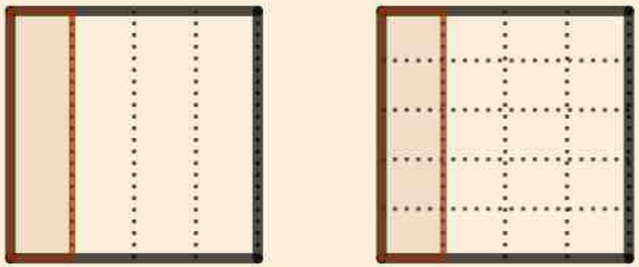
because:

$\frac{1}{4}$ has been expanded

by the number 5.

Correct!

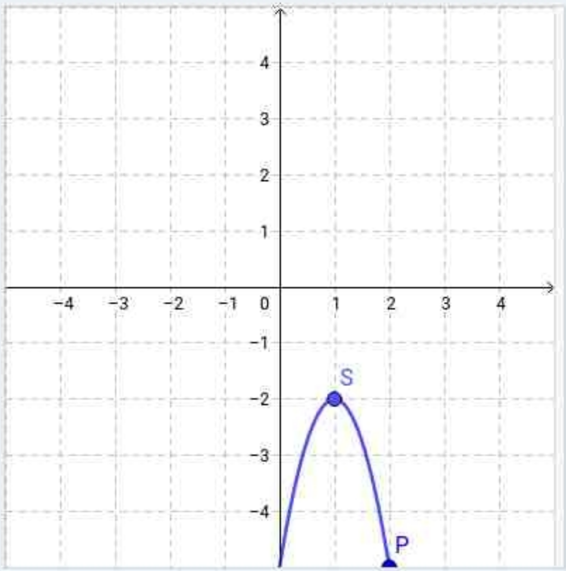
Because, as you can see,



$\frac{1}{4} \overset{\cdot 5}{\underset{\cdot 5}{=}} \frac{5}{20}$

examples

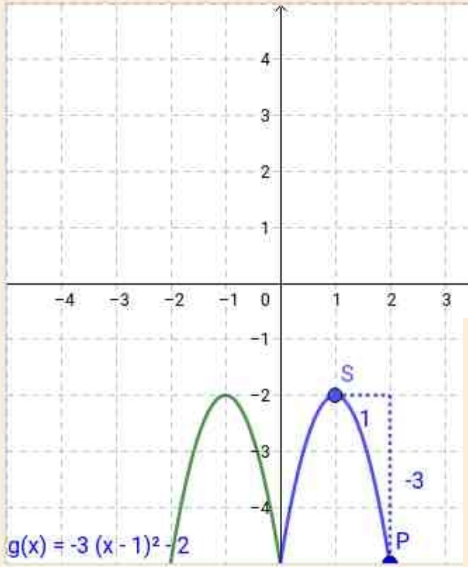
Move the points S und P,
such that the graph fits with
$f(x) = -3 \cdot (x+1)^2 - 2.$



Check

Wrong, too bad!

The green graph would be correct.



Why?

You can find out yourself.

Correct your blue graph
and watch how the expression changes.

Try to find answers to the following questions:

1. Where in the expression
   can you see the coordinates of the vertex?

2. Where in the expression
   can you see a value for the opening
   of the parabola?

Do you have an idea already?

Then try the task again.

Or wait 30 seconds
after which a full solution will appear:

Musterlösung

overview

1. examples
2. theory
3. suggestions

a short digression
into
AuthOMath

Co-founded by the European Union
2021-1-DE01-KA220-HED-000032031

AuthOMath

AuTo

- a moodle based authoring tool
  for randomized interactive and dynamic multimodal mathematical tasks
  with automatic adaptive feedback

DiCo

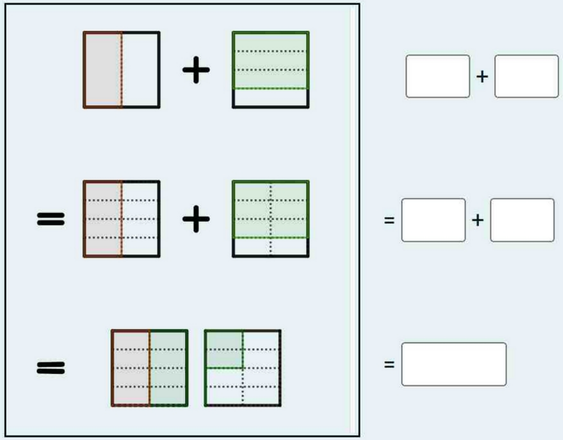
- a didactical concept
  for designing online based interactive learning material
  for use in mathematics teacher education

Co-founded by the European Union
2021-1-DE01-KA220-HED-000032031

AuthOMath

This visualization shows
how two fractions are added.

Translate into maths:



```
n1:rand([2,3,4,5]);
n2:rand_with_prohib(2,5,[n1]);
z1:rand(n1-1)+1;
z2:rand(n2-1)+1;
```

```
<br>This visualization shows<br>how two fractions are added.<br><br>Tra

<table style="border-collapse: collapse; width: 400px;" border="0">
    <tbody>
        <tr>
            <td style="width: 180px; vertical-align: bottom; border-sty

                [[geogebra set="n1,n2,z1,z2,x1,y1,x2,y2"]]
                params["material_id"] = "yqhjpr2c";
                params["width"] = 450;
                params["height"] = 550;
                params["borderColor"] = "rgba(0, 0, 0, 0)";
                params["transparentGraphics"]= true;
                params["scale"] = 0.5;
                [[/geogebra]]

            </td>
```

AuthOMath

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```

names of variables in applet, with

- set: transmit values from STACK to applet
- watch: read values from applet into STACK on “Check”
- remember: remember values for reloading applet

```
>This visualization shows<br>how two fractions are added.<br><br>Tra

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            </td>
```

applet ID on geogebra.org

GeoGebra App Parameters

https://wiki.geogebra.org/en/Reference:GeoGebra_App_Parameters

overview

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for more on AuthOMath,

cf. www.authomath.org

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2021-1-DE01-KA220-HED-000032031

feedback

...is information
about performance

...its function is
assisting learning

...hence should
be perceived as
advice for action

Ras, Whitelock & Kalz (2016): The promise and potential of e- assessment for learning◦ Black & Wiliam (2009): Developing the theory of formative assessment ◦
Hattie & Timperley (2007): The Power of Feedback ◦ Kluger & DeNisi (1996) The Effects of Feedback Interventions on Performance

parameters

width of focus

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focus : idea

- from procedures

  to underlying concepts

- from addressing procedures that are necessary to master the given task

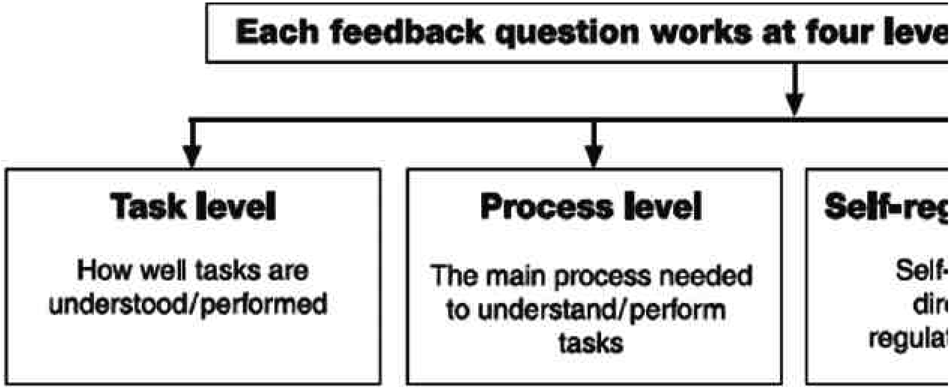
  to providing the conceptual basis for understanding the given and related tasks

Weigand, Schüler-Meyer & Pinkernell (2022): Didaktik der Algebra ∘ Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ Wolff (2018): Umformen und Lösen von quadratischen Gleichungen ∘ vom Hofe & Blum (2016): “Grundvorstellungen” as a Category of Subject-Matter Didactics ∘ Rittle-Johnson & Star (2009): Compared with what? ∘ Black & Wiliam (2009): Developing the theory of formative assessment ∘ Prediger (2008): Discontinuities for mental models

focus : idea

- from procedures

  to underlying concepts



Hattie & Timperley (2007): The Power of Feedback

focus

- from procedures

  to underlying concepts

Calculate:

$\frac{1}{2} + \frac{1}{5} =$ 2/10

Wrong, sorry!

You have found a common denominator.

But also expand the numerators:

$$\frac{1}{2} + \frac{1}{5} = \frac{1 \cdot 5}{2 \cdot 5} + \frac{1 \cdot 2}{5 \cdot 2}$$

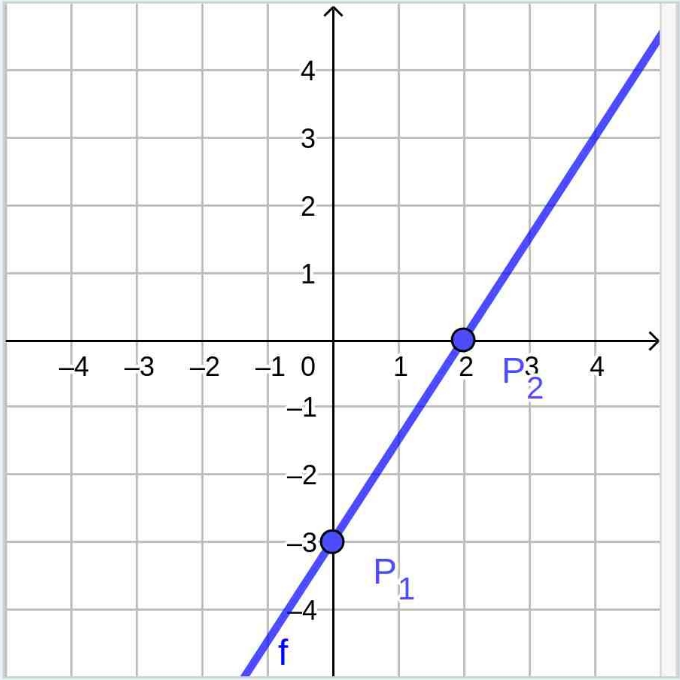
focus

- from procedures

  to underlying concepts

Give the graph to the function
$f(x) = 2 \cdot x - 3$.

Place P_1 and P_2
such that the line fits the expression.



Follow these steps:

1. Place P_1

The number -3 in $f(x) = 2 \cdot x - 3$
marks the place on the y-axis.

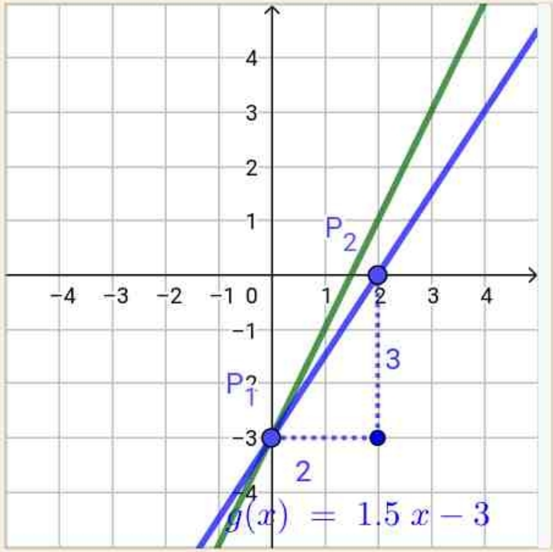
Place P_1 here.

2. Place P_2

The other number 2 in $2 \cdot x - 3$
denotes the slope of the line.

Hence start with P_2 in P_1,
then move P_2 one step to the right,
and after that move 2 steps vertically.

Place P_2 here.



focus

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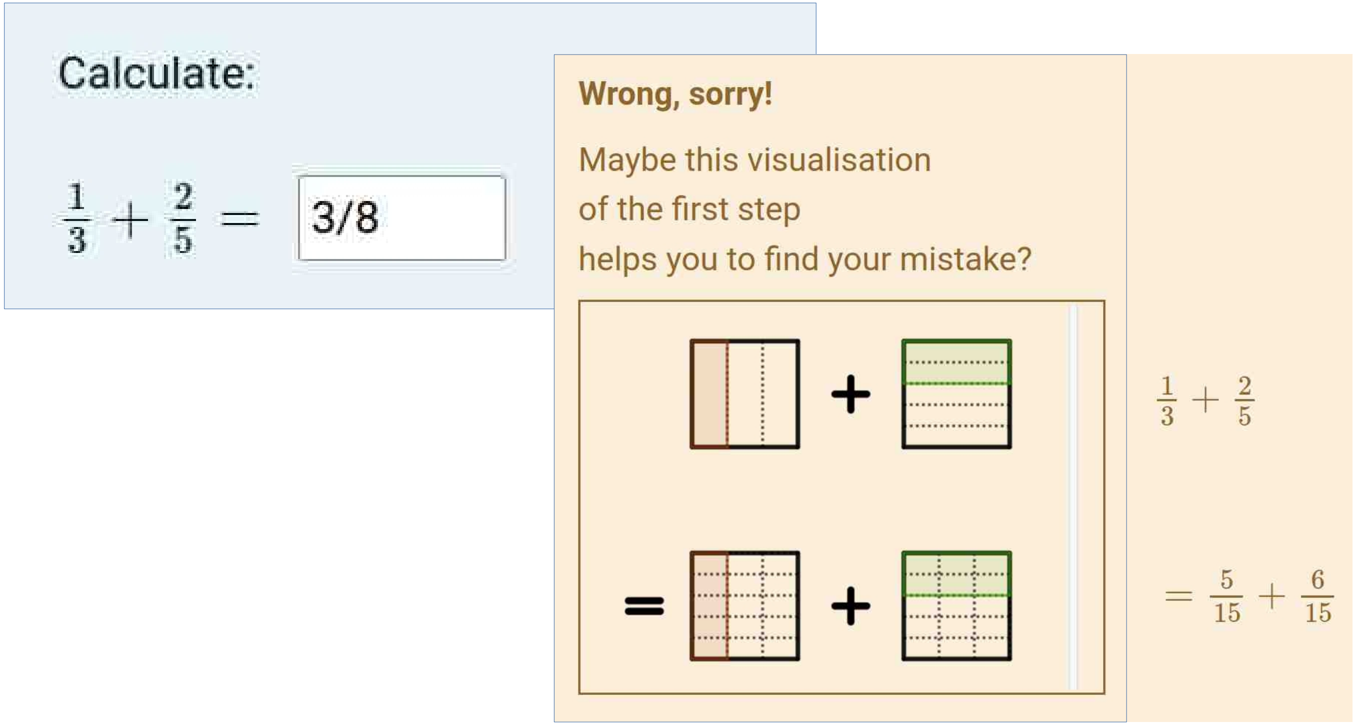


focus

- from procedures

  to underlying concepts

Solve:

$2\cdot(q+1)=4$

Copy the equation below,
then note each next step beneath:

2*(q+1)=4
2*q+2=4
2q=2
q=1

$L = \{$ 1 $\}$

Good. Your solution is correct.
And the transformations are fine.

But that took long!

There is a faster solution - compare:

$2\cdot(q+1)=4$ $2\cdot q+2=4$ …	$2\cdot(q+1)=4$ $q+1=2$ …

One is your strategy,
the other is faster.

Well? Do you have an ide

Then try this task again.

Else wait for 30 sec,
then a full solution appea

Click here for full solutio

Here you have two full solutions:

Both are correct.

$2\cdot(q+1)=4$ $2\cdot q+2=4$ $2\cdot q=2$ $q=1$	$2\cdot(q+1)=4$ $q+1=2$ $q=1$

Choose the more efficient and
try again!

Try another question like this one

Weigand, Schüler-Meyer & Pinkernell (2022): Didaktik der Algebra ∘ Wolff (2018): Umformen und Lösen von quadratischen Gleichungen ∘ Rüede (2013): How secondary level teachers and students impose personal structure on fractional expressions and equations ∘ Rittle-Johnson & Star (2009): Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving.

focus : feedback content

- from
  procedures

  to underlying
  concepts

- worked out
  solving procedure
- specific reference
  to single steps
- interactive scaffolding
  through steps
- references
  to relevant rules

- explanatory models
  (“Grundvorstellungen”)
- representational
  or contextual flexibility
  (e.g. geometric visualisations,
  numeric examples,
  familiar contexts
  from outside maths,
  if not part of the task)
- strategic flexibility

Weigand, Schüler-Meyer & Pinkernell (2022): Didaktik der Algebra ∘ Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ Wolff (2018): Umformen und Lösen von quadratischen Gleichungen ∘ vom Hofe & Blum (2016): “Grundvorstellungen” as a Category of Subject-Matter Didactics ∘ Rüede (2013): How secondary level teachers and students impose personal structure on fractional expressions and equations ∘ Rittle-Johnson & Star (2009): Compared with what? ∘ Prediger (2008): Discontinuities for mental models

focus : think about it

- from procedures

  to underlying concepts

"a deep understanding of learning involves the construction of meaning (understanding) and relates more to the relationships, cognitive processes, and transference to other more difficult or untried tasks" (Hattie & Timperley, 2007)

mastery of procedures reduce cognitive load while solving complex and challenging problems

Weigand, Schüler-Meyer & Pinkernell (2022): Didaktik der Algebra ∘ Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ Wolff (2018): Umformen und Lösen von quadratischen Gleichungen ∘ vom Hofe & Blum (2016): "Grundvorstellungen" as a Category of Subject-Matter Didactics ∘ Rittle-Johnson & Star (2009): Compared with what? ∘ Prediger (2008): Discontinuities for mental models

parameters

width of focus
grade of adaption
grade of activation

adaption : idea

- from nearly none

  to very differen-tiating

- the same feedback regardless what the (wrong) answer is

  specific feedback for each answer case

adaption

- from nearly none

  to very differen-tiating

Give a cubic expression
which has exactly the two roots 1 und 4 .

$f(x) =$ (x-4)*(x-1)

Wrong, too bad.

A correct expression would be $(x-4)^2 \cdot (x-1)$.

Why is that?

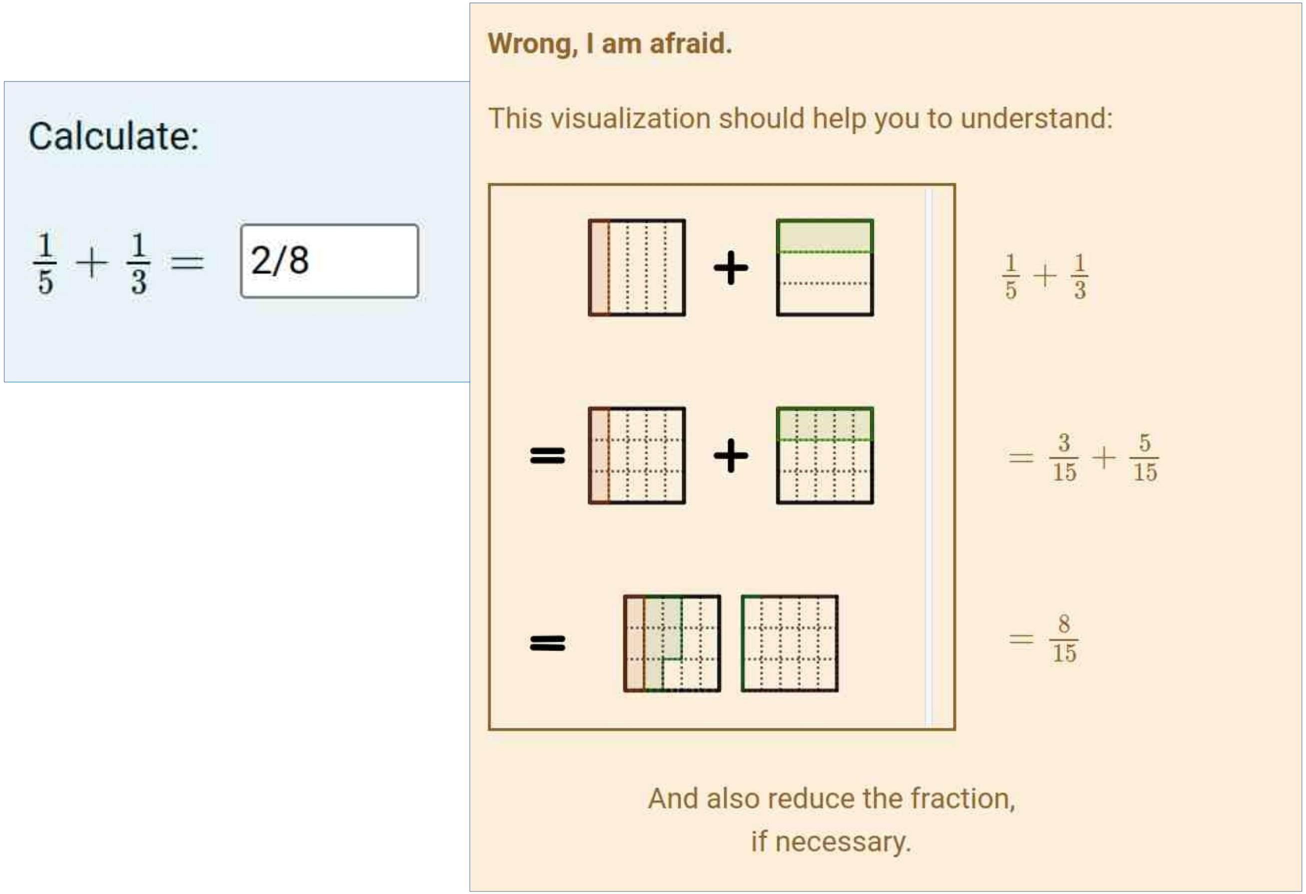
You need to know

that a linear expression like $(x-a)$ has a as root,
that $(x-a) \cdot (x-b)$ is a quadratic expression and has a und b as roots,
and that $(x-a) \cdot (x-b) \cdot (x-c)$ is a cubic expression with roots a, b und c.

adaption

- from nearly none

  to very differen-tiating



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Calculate:

$\frac{1}{2} + \frac{1}{5} =$ 2/10

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But also expand the numerators:

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adaption

- from nearly none

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Give a quadratic expression
which has exactly the two roots -3 und -1 .

$f(x) =$ (x-3)*(x-1)

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer again...

adaption : content

- from nearly none

  to very differen-tiating

- basic procedural and/or conceptual knowledge for mastering all varieties of the task

- specific advice (procedural or conceptual) for a priori identified answer cases:

  correct, different in strategies

  wrong, different as to systematic errors or misconceptions

adaption : think about it

- from nearly none

  to very differen-tiating

adaption supports acceptance and certainty about how to proceed

in retention tasks, specific feedback is superior to general advice.

in transfer tasks, no difference between specific and general advice

Ras, Whitelock & Kalz (2016): The promise and potential of e- assessment for learning∘ Shute (2008): Focus on Formative Feedback ∘ Phye & Sanders (1994): Advice and Feedback: Elements of Practise For Problem Solving

parameters

width of focus

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activation : idea

- from
  receptive

  to
  active

- from informing
  about (parts of) the
  necessary knowledge

  to prompting the learner
  to (re)construct
  the necessary knowledge
  by him/herself

Hattie & Timperley (2007): The Power of Feedback ∘ Kluger & DeNisi (1996) The Effects of Feedback Interventions on Performance

activation

- from receptive

  to active

As you know,

$$(a+b)^2 = a^2 + 2\cdot a\cdot b + b^2$$
$$(a-b)^2 = a^2 - 2\cdot a\cdot b + b^2$$
$$(a-b)\cdot(a+b) = a^2 - b^2$$

Now factorise $18\cdot s^2 + 24\cdot s\cdot t + 8\cdot t^2$ by using one of the three formulas above.

You can do your calculations here:

```
18*s^2+24*s*t+8*t^2
= (18*s+8*t)^2
```

Denote your solution here:

```
(18*s+8*t)^2
```

Wrong, too bad.

Correct would be $2\cdot(3\cdot s+2\cdot t)^2$

That's how to do it:

Here is the expression again:

$18\cdot s^2 + 24\cdot s\cdot t + 8\cdot t^2$

First, you need to find two square numbers.
You can identify them once you factor out 2:

$= 2\cdot(9\cdot s^2 + 12\cdot s\cdot t + 4\cdot t^2)$

Now the square numbers are visible inside the brackets: 9 und 4

Second, choose from the three formulas mentioned above the one that has the same structure as the expression inside the brackets:

$9\cdot s^2 + 12\cdot s\cdot t + 4\cdot t^2$
corresponds to
$a^2 + 2\cdot a\cdot b + b^2$

Third, identify the corresponding parts of each expression:

a^2 corresponds to $9\cdot s^2$, hence $a = 3\cdot s$, and
b^2 corresponds to $4\cdot t^2$. So $b = 2\cdot t$

And check whether $2\cdot a\cdot b$ corresponds to $12\cdot s\cdot t$:
$2\cdot 3\cdot s\cdot 2\cdot t = 12\cdot s\cdot t$,
which hence is the case.

Fourth, substitute the values for a and b in $(a+b)^2$.
And do not forget the factor from the first step to denote the final solution:

$= 2\cdot(3\cdot s+2\cdot t)^2$

Try this task again!!

Hoch & Dreyfus (2005): Students' Difficulties with Applying a Familiar Formula in an Unfa

activation

- from receptive

  to active

Write $\frac{3}{4}$ as a decimal number.

$\frac{3}{4}$ = 3.4

Tip

Follow these steps

1. Expand the fraction s
2. Count the number of
3. Formulate the decima

Do you know now what to d

Change your solution above
and click on "check".

Else wait 30 sec
for "more help" below.

more help

Write $\frac{3}{4}$ as a decimal number.

Fill the blanks:

1. Expand the fraction such that the denominator is 10 or 100 or 1000...

Expand $\frac{3}{4}$ by ____

$= \frac{3 \cdot 25}{4 \cdot 25}$

= ____ (enter a fraction here)

Count the number of zeros of the new denominator.

The denominator of $\frac{75}{100}$ has ____ zero(s). (enter a number here)

3. Formulate the decimal number

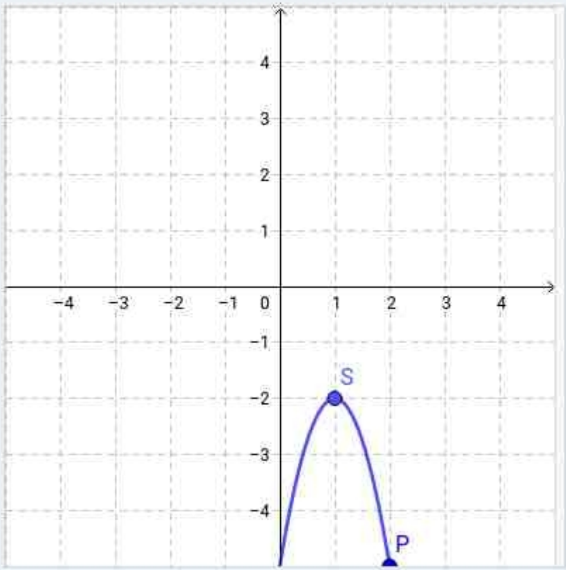
$\frac{75}{100}$ in the form of a decimal number: ____

activation

- from receptive

  to active

Move the points S und P,
such that the graph fits with
$f(x) = -3 \cdot (x+1)^2 - 2.$



Check

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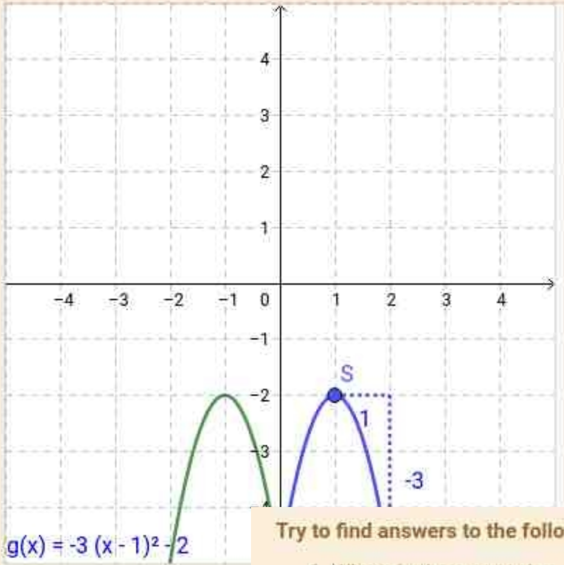
The green graph would be correct.



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Or wait 30 seconds
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Musterlösung

activation

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Give a quadratic expression
which has exactly the two roots -3 und -1 .

$f(x) =$ (x-3)*(x-1)

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer again...

activation : content

- from
  receptive

  to
  active

- statements,
  propositions,
  description
- pictures,
  graphs
- videos,
  movies

- clozes,
  scaffolding

- questions,
  hints,
  food for thought
- interactive
  elements
  for exploration

activation : think about it

- from receptive

  to active

“Interactive feedback is more effective than other kinds of feedback in improving students’ performance.”

“Unless students see themselves as agents of their own change, and develop an identity as a productive learner who can drive their own learning, they may neither be receptive to useful information about their work, nor be able to use it.”

for experts, corrective or thought provoking feedback seems sufficient

for novices, scaffolding or worked out examples are needed

Barana et al. (2021): Boud & Molloy (2013): Kirschner, Sweller & Clark, 2006; Renkl, 2002; Renkl & Atkinson, 2003 Pinkernell et al., 2020

parameters

width of focus

grade of adaption

grade of activation

...and structure

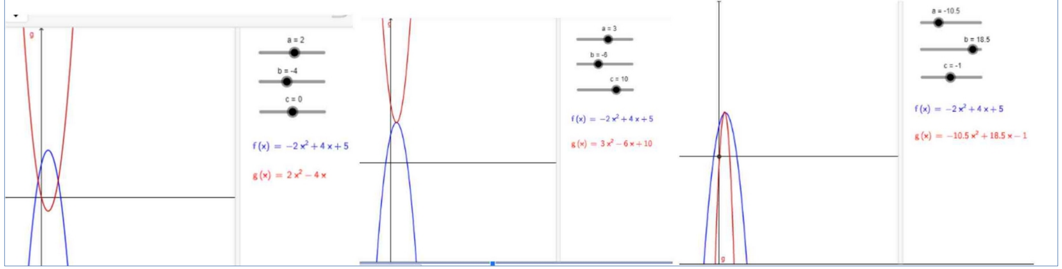
structure

location
order
timing

- as part of task

Create three example sets of functions f and g
that follow as many of the following conditions as possible:

(1) the graph of f(x) intersects the graph of g(x)
in exactly one point;
(2) the two functions have the same symmetry axes;
(3) g(x) passes through the origin (0,0) of the system; and
(4) the function g(x) has a minimum.

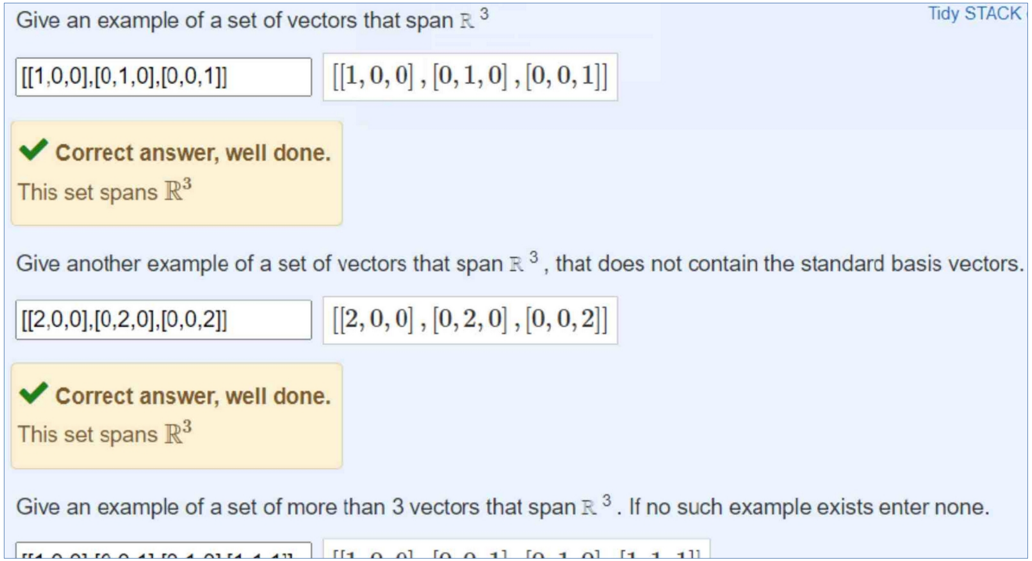


Yerushalmy (2019): Seeing the Entire Picture (STEP):

structure

location
order
timing

- as part of task

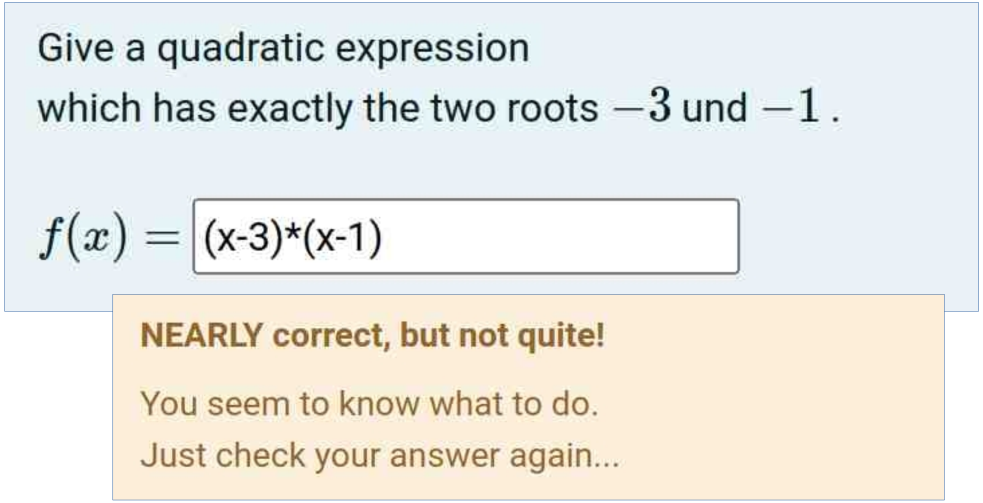


Kinnear & Foster (2001): How can e-assessments be designed to expand and enrich students' example spaces? ∘ Fahlgren et al. (2023): Example-generating tasks in a computer-aided assessment system ∘ Kinnear (2023): Student approaches to generating mathematical examples

structure

location
order
timing

- as part of task
- immediately after task



structure

location
order
timing

- as part of task
- immediately after task
- delayed (in bits)

Give a quadratic expression
which has exactly the two roots -3 und -1.

$f(x) =$ (x-3)*(x-1)

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer again...

Here is how:

You need to know:
An expression like $(x-a)\cdot(x-b)$ is quadratic and has a and b as roots.

To have -3 and -1 as roots
$(x+1)\cdot(x+3)$ would fit.

Try again!

structure

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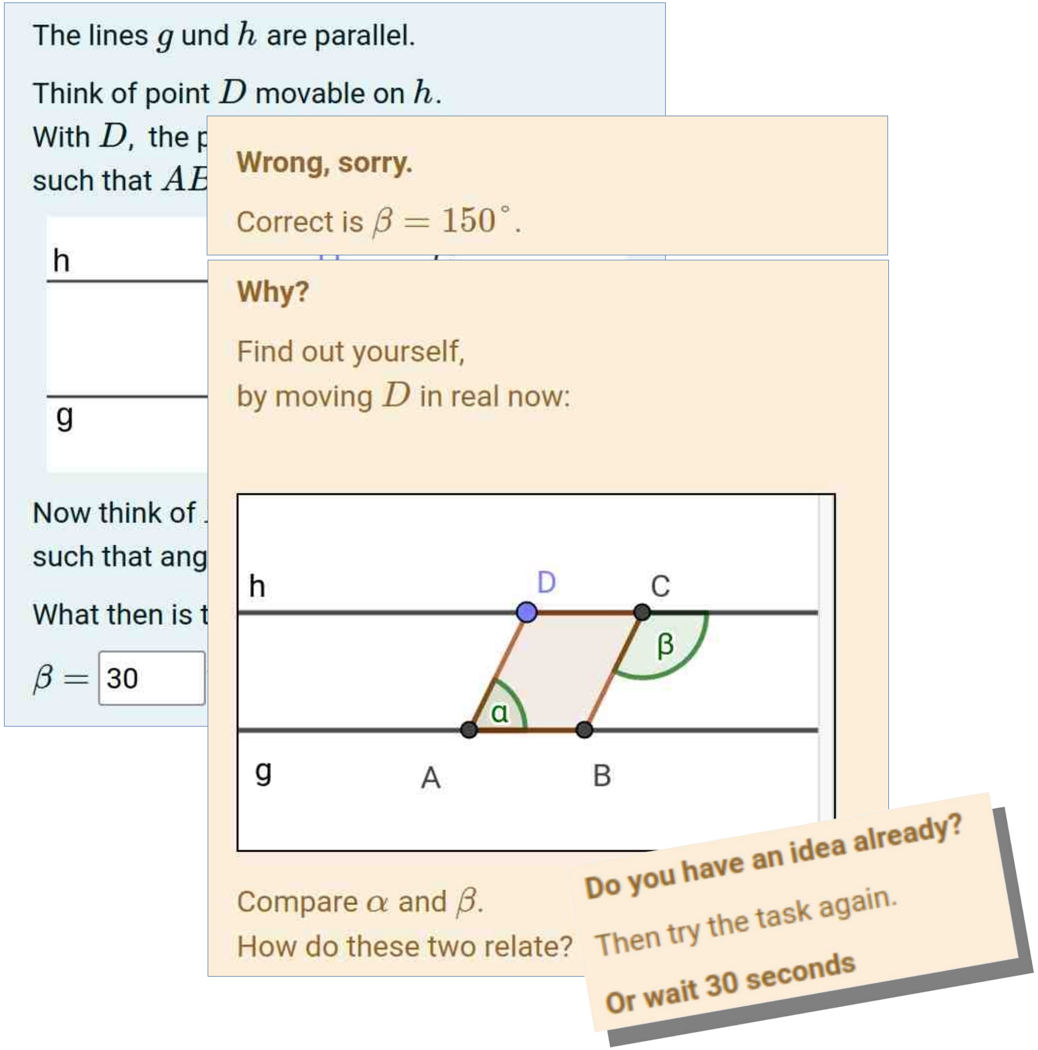
order
timing

- as part of task
- immediately after task
- delayed (in bits)



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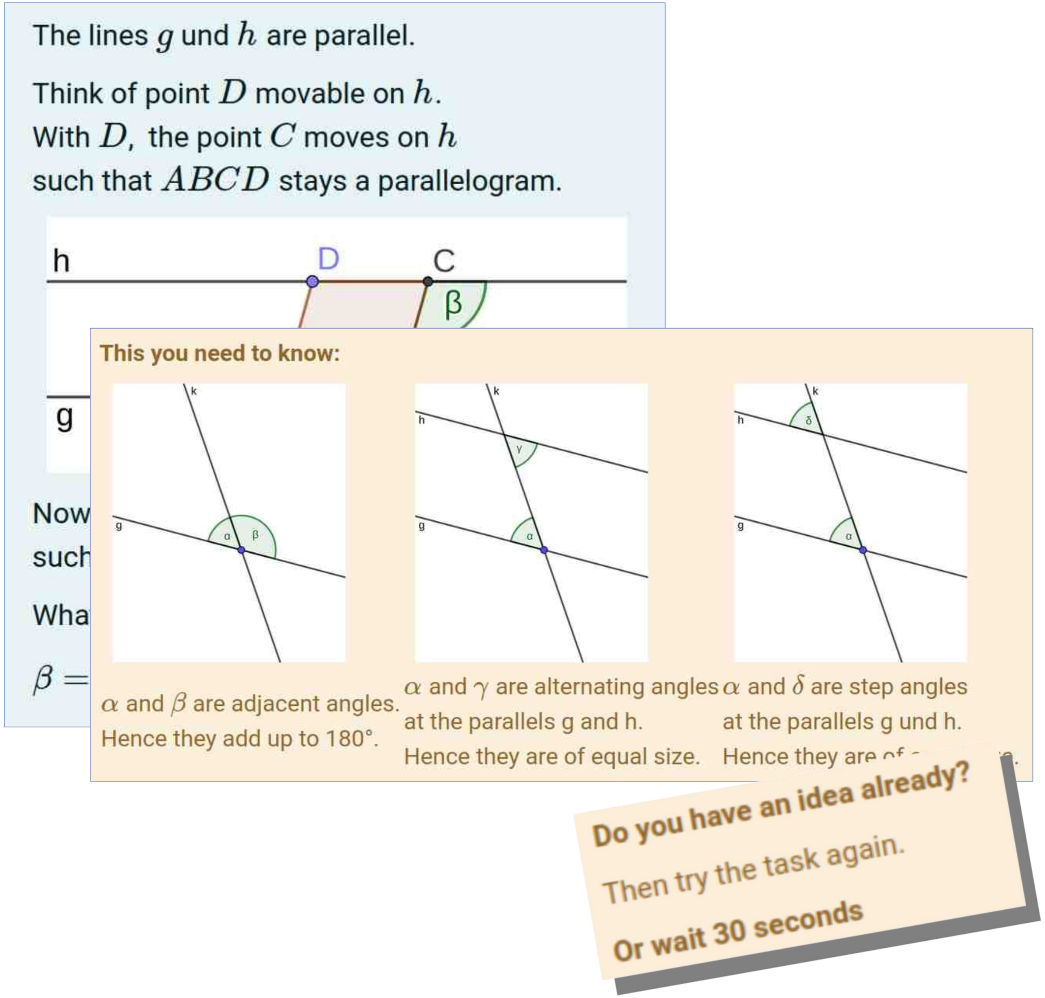

structure

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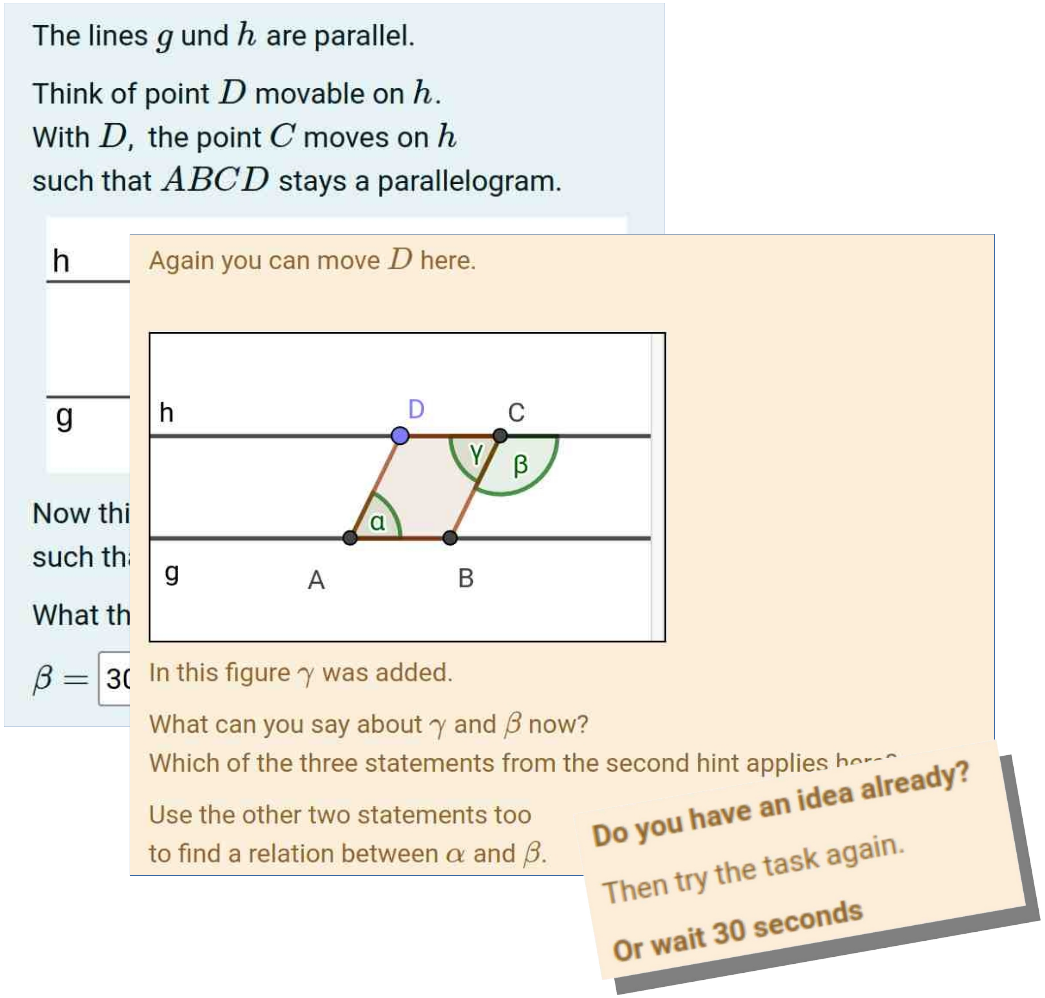

structure

location
order
timing

- as part of task
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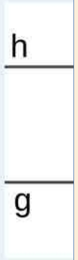
- delayed (in bits)

The lines g und h are parallel.

Think of point D movable on h.

With D …

such th…



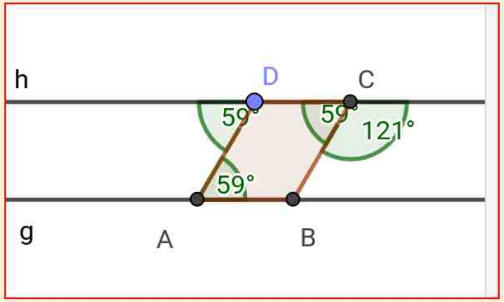
Now th…

such th…

What t…

$\beta =$ 3…

Move the point D now.



And you can see that

1: angels at A und D are always of equal size,
2: both angels at D and the inner angel at C are always at equal size,
3: both angels at C add up to 180°.

Why is that?

1 is correct because both angles are alternating angles at the parallels g and h,
2 is correct since both angels are step angles at the parallels g and h, and
3 is correct because both angles are adjacent angels.

Hence both angles at A and the exterior angle at C add up to 180°.

In short:

$\beta = 180^\circ - \alpha = 180^\circ - 30^\circ = 150^\circ$

structure

location
order
timing

"Give a moment to think it over…"

for low achievers, prompt timing, for high achievers, delayed timing of feedback seems suitable

when testing declarative knowledge feedback only after second try is more effective

Shute (2008): Focus on Formative Feedback ∘ Richards (1989): A comparison of three computer-generated feed-back strategies ∘ Mory (2004): Feedback Research Revisited

overview

1. examples
2. theory
3. suggestions

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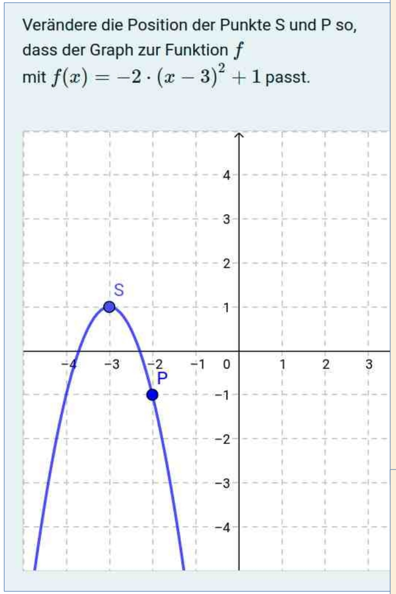
models

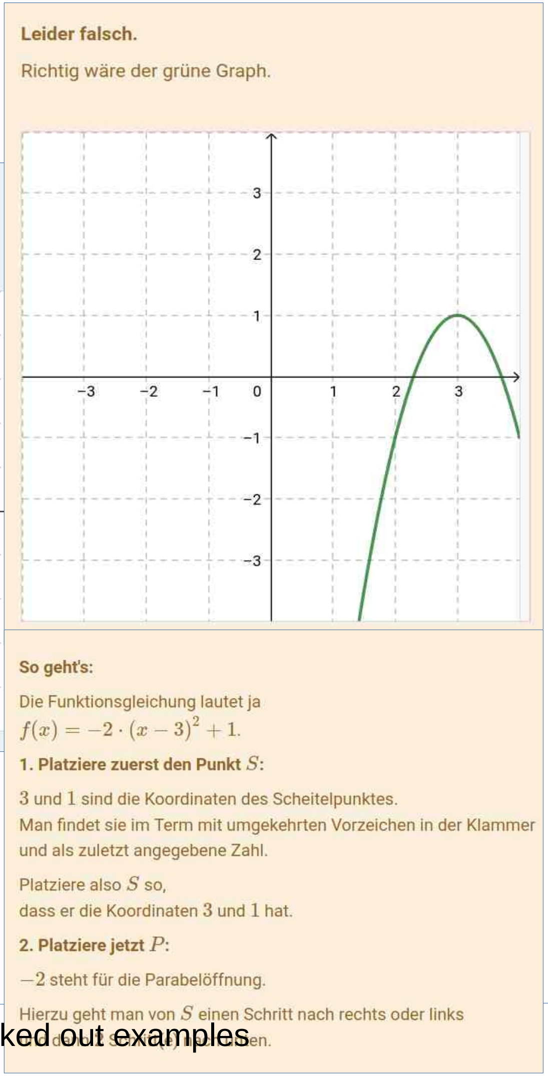
worked solution

Sorry, wrong (KR)
Correct would be… (KCR)
This is how to do it correctly: … (KH)

appears without delay

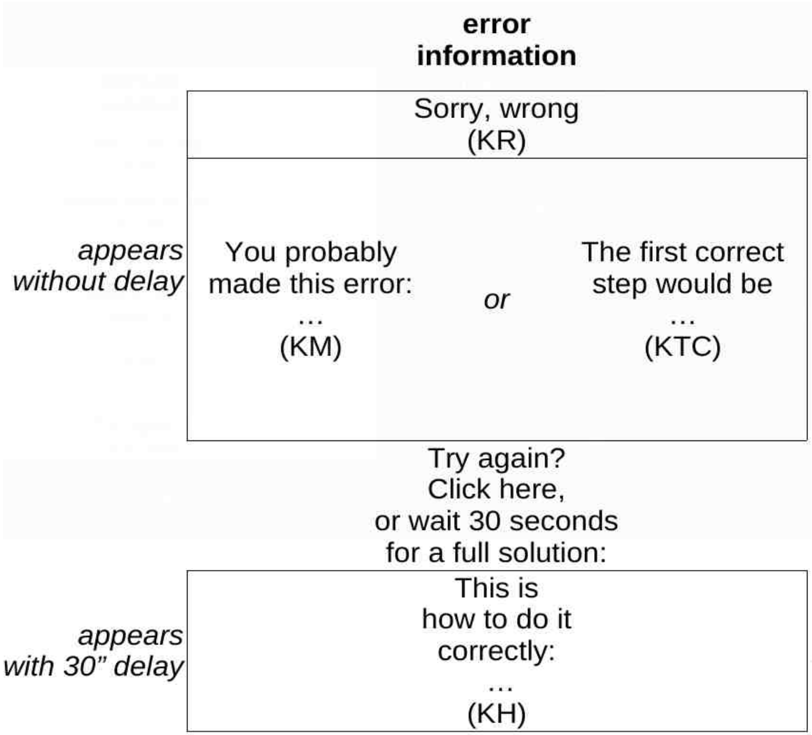
Try again?
Click here:

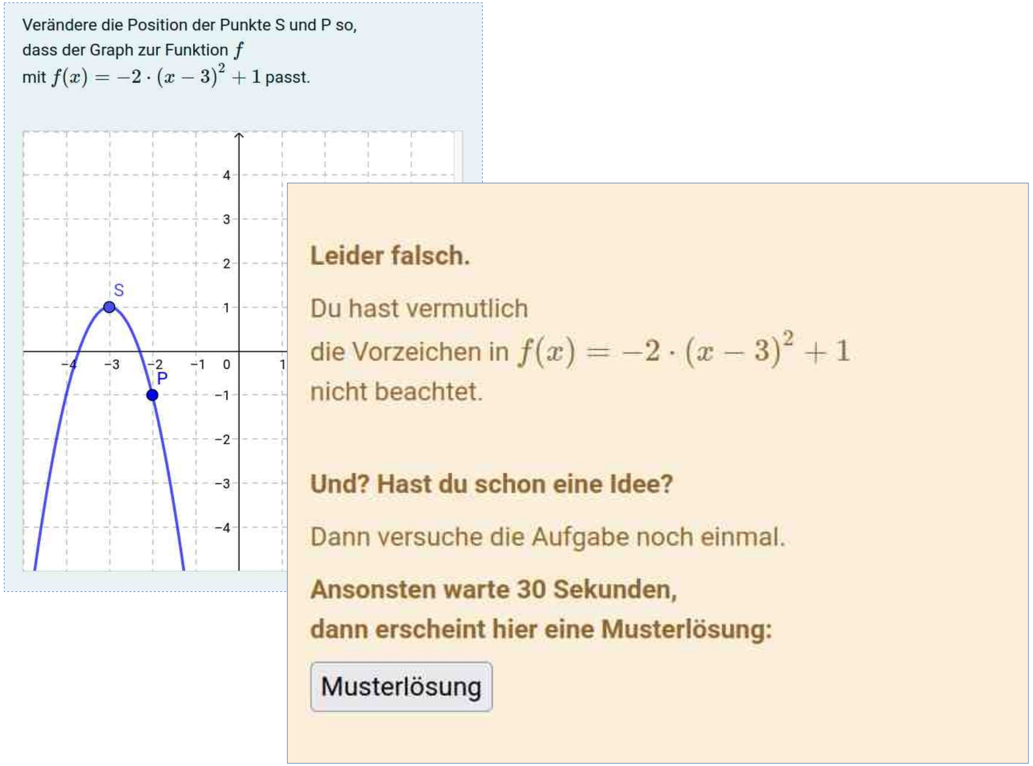




Pinkernell, Gulden & Kalz (2020) : Automated feedback at task level: error analysis or worked out examples

models





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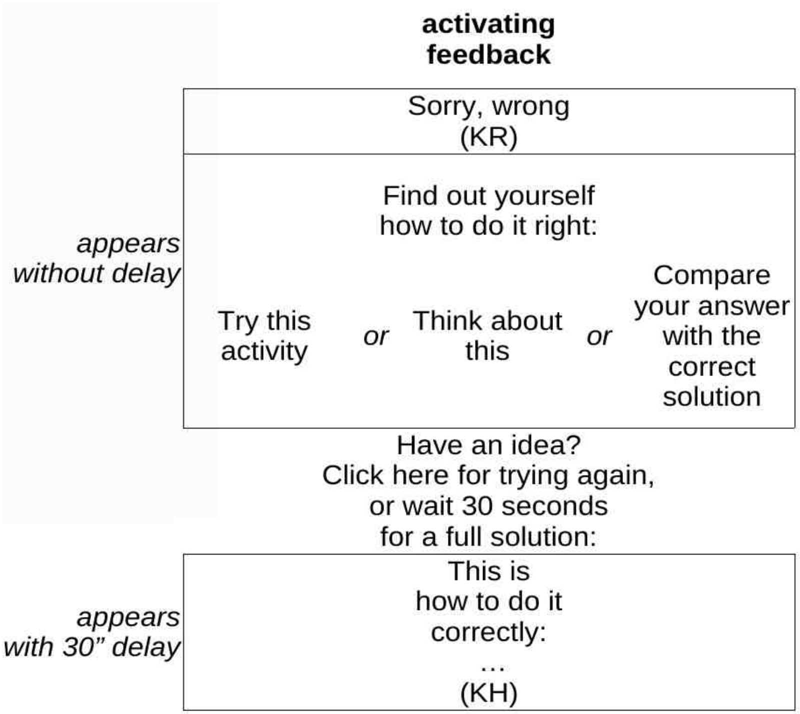
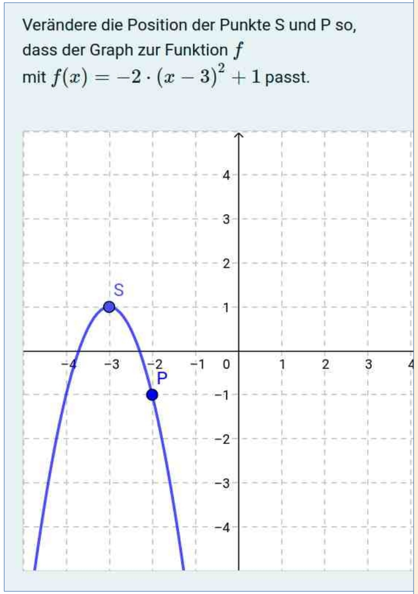
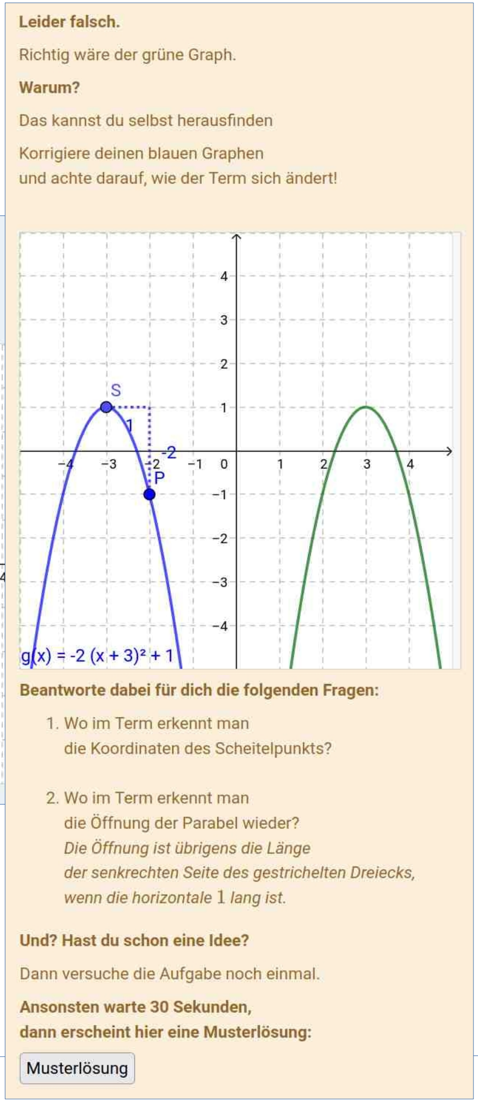
models


activating feedback
Sorry, wrong
(KR)
appears
without delay
Find out yourself
how to do it right:
Try this
activity
or
Think about
this
or
Compare
your answer
with the
correct
solution
Have an idea?
Click here for trying again,
or wait 30 seconds
for a full solution:
appears
with 30" delay
This is
how to do it
correctly:
…
(KH)


Verändere die Position der Punkte S und P so,
dass der Graph zur Funktion f
mit $f(x) = -2 \cdot (x-3)^2 + 1$ passt.
S
P


Leider falsch.
Richtig wäre der grüne Graph.
Warum?
Das kannst du selbst herausfinden
Korrigiere deinen blauen Graphen
und achte darauf, wie der Term sich ändert!
S
P
g(x) = -2 (x + 3)² + 1
Beantworte dabei für dich die folgenden Fragen:
1. Wo im Term erkennt man
die Koordinaten des Scheitelpunkts?
2. Wo im Term erkennt man
die Öffnung der Parabel wieder?
Die Öffnung ist übrigens die Länge
der senkrechten Seite des gestrichelten Dreiecks,
wenn die horizontale 1 lang ist.
Und? Hast du schon eine Idee?
Dann versuche die Aufgabe noch einmal.
Ansonsten warte 30 Sekunden,
dann erscheint hier eine Musterlösung:
Musterlösung

models

feedback
referring to explanatory models

Sorry, wrong (KR)
This helps you to understand the concept: [explanatory model]

appears without delay

Have an idea?
Click here for trying again,
or wait 30 seconds
for a full solution:

This is how to do it correctly: … (KH)

appears with 30” delay

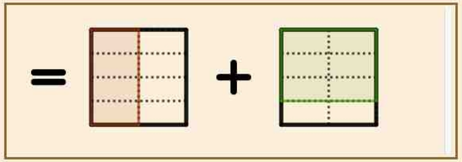
Calculate:

$\frac{1}{2} + \frac{3}{4} =$ 4/6

Too bad, not fully correct.

Why is that?

Maybe this translation of the second line
gives you an idea?



$\frac{4}{8} + \frac{6}{8}$

Do you know what to do now?

Then reload another question
and try again.

Or wait for 15 seconds for a full solution:

Click here for a full solution.

summary

1. examples
2. theory
3. suggestions

- width of feedback focus
  on procedural or conceptual knowledge
- grade of adaption
  to student correct or wrong answers
- grade of activation
  to foster change from receptive to active attitude
- structure and timing
  to model sensible learning paths

- model 1: full solution
- model 2: error information
- model 3: activating feedback
- model 4: reference to explanatory models

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