Activating Elements in Formative Feedback: From Receptive to Active Learning with Automated Feedback

Guido Pinkernell ∘ Pädagogische Hochschule Heidelberg
STACK Community Meeting OTH Amberg ∘ March 2024

overview

1. what this is about
2. some background theory
3. activating feedback
4. feedback design parameter

The creation of these resources has been (partially) funded by the ERASMUS+ grant program of the European Union under grant No. 2021-1-DE01-KA220-HED-000032031. Neither the European Commission nor the project's national funding agency DAAD are responsible for the content or liable for any losses or damage resulting of the use of these resources.

Give a quadratic exp
which has exactly the

$f(x) =$ (x-5)*(x-3)

mind the signs

check by substitu-tion

quite good

$\frac{3}{4} = \frac{9}{12}$

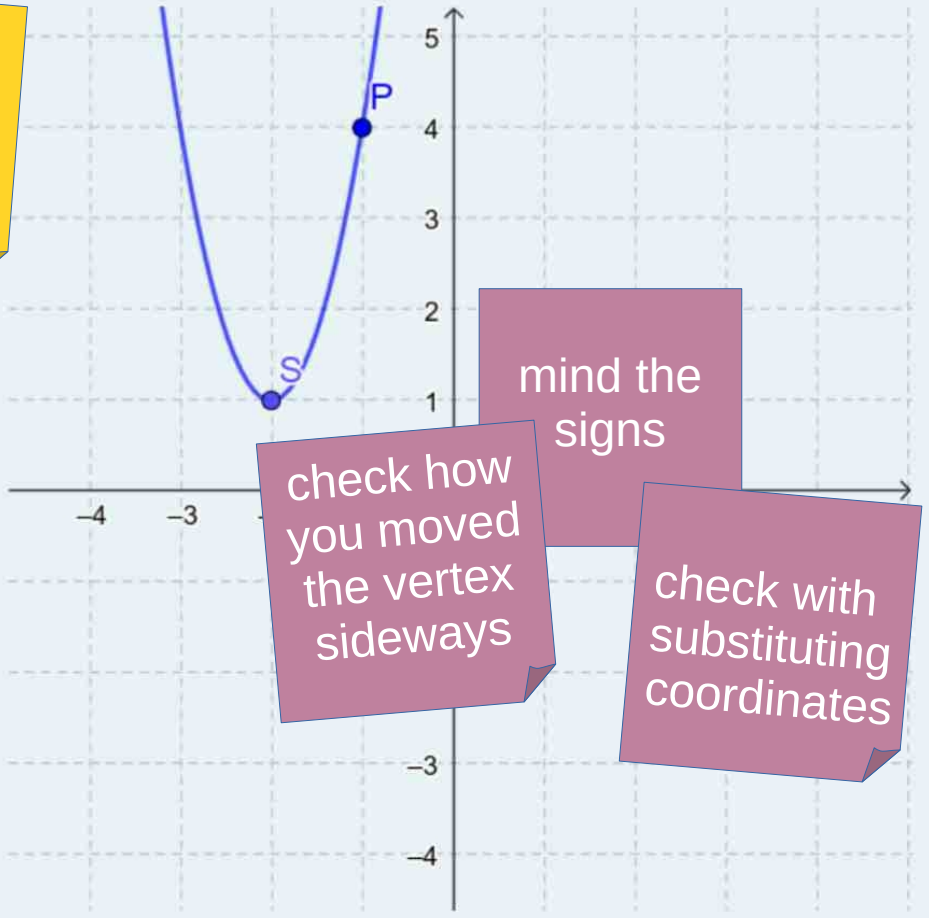
because:

$\frac{3}{4}$ has been expanded

by the number 3 .

correct

Move the points S und P,
such that the graph fits with
$f(x) = 3 \cdot (x-2)^2 + 1.$



mind the signs

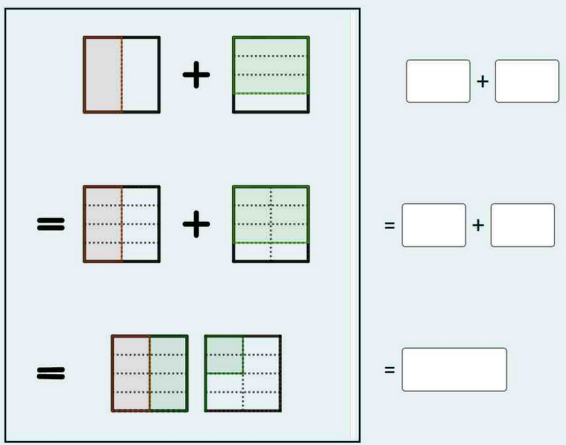
check how you moved the vertex sideways

check with substituting coordinates

what this is about

This visualization shows
how two fractions are added.

Translate into maths:



```
n1:rand([2,3,4,5]);
n2:rand_with_prohib(2,5,[n1]);
z1:rand(n1-1)+1;
z2:rand(n2-1)+1;
```

```
1  <br>This visualization shows<br>how two fractions are added.<br><br>Tra
2
3  <table style="border-collapse: collapse; width: 400px;" border="0">
4      <tbody>
5          <tr>
6              <td style="width: 180px; vertical-align: bottom; border-sty
7
8                  [[geogebra set="n1,n2,z1,z2,x1,y1,x2,y2"]]
9                  params["material_id"] = "yqhjpr2c";
10                 params["width"] = 450;
11                 params["height"] = 550;
12                 params["borderColor"] = "rgba(0, 0, 0, 0)";
13                 params["transparentGraphics"]= true;
14                 params["scale"] = 0.5;
15                 [[/geogebra]]
16
17             </td>
```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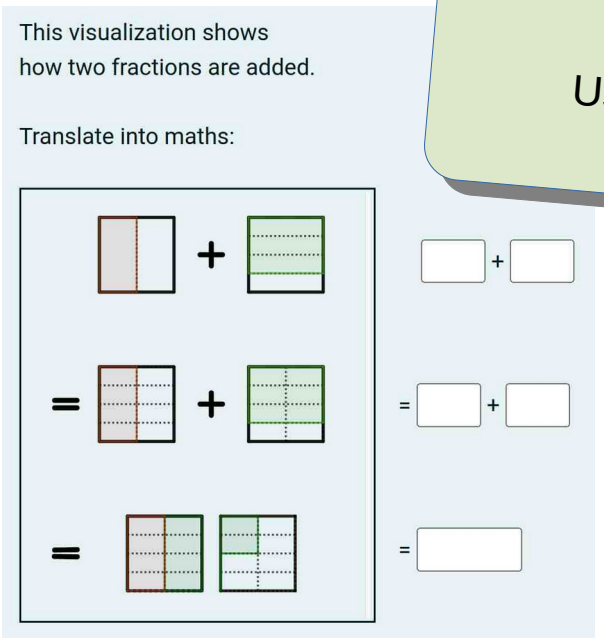
what this is about

Tuesday 11 – 12:30

Workshop

AuthOMath event:
Using GeoGebra in STACK




```
n1:rand([2,3,4,5]);
```

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15                 [[/geogebra]]
16
17             </td>
```


www.authomath.org
mathebruecke.pinkernell.online

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what this is about

Idea combining strengths and communities of GeoGebra and STACK

- interactivity
- dynamization
- multimodality

- adaptive feedback
- randomization

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cognitive activation

in general
teaching

in math
teaching

in math
tasks

in math task
feedback

cognitive activation

in general teaching

in math teaching

in math tasks

in math task feedback

- one of three basic dimensions of teaching quality
  - classroom management
  - student orientation
  - cognitive activation
- means activities and scenarios that foster autonomous thinking in learners while learning

Klieme, E., & Rakoczy, K. (2003). Unterrichtsqualität aus Schülerperspektive: Kulturspezifische Profile, regionale Unterschiede und Zusammenhänge mit Effekten von Unterricht. ∘ Praetorius, A.-K., Klieme, E., Herbert, B., & Pinger, P. (2018). Generic dimensions of teaching quality: The German framework of Three Basic Dimensions

cognitive activation

in general teaching

in math teaching

in math tasks

in math task feedback

- characteristics
  - challenging tasks and questions
  - activating prior knowledge
  - exploration the students' ways of thinking
  - avoid receptive understanding
  - discursive and co-constructive learning
  - genetic-socratic teaching
  - supporting metacognition
- significant praedictor for mathematics achievement

Praetorius, A.-K., Klieme, E., Herbert, B., & Pinger, P. (2018). Generic dimensions of teaching quality: The German framework of Three Basic Dimensions◦ Merk, S., Batzel-Kremer, A., Bohl, T., Kleinknecht, M., & Leuders, T. (2021). Nutzung und Wirkung eines kognitiv aktivierenden Unterrichts bei nicht-gymnasialen Schülerinnen und Schülern. Unterrichtswissenschaft.

cognitive activation

in general teaching

in math teaching

in math tasks

in math task feedback

- aus Merkmalsangaben, anwendbar für aktivierendes Feedback-Design
- keine
  - kleinschrittige oder genaue Anweisungen zur Lösung von Aufgaben
  - unmittelbare Information über richtig oder falsch
- sondern
  - offene Fragen, die zum Nachdenken anregen
  - Aufforderungen zur Begründung von
  - oder darüber nachzudenken, was sich aus den Antworten ergibt
- Vergleich und Bewertung verschiedener Aufgabenlösungen

Praetorius, A.-K., Klieme, E., Herbert, B., & Pinger, P. (2018). Generic dimensions of teaching quality: The German framework of Three Basic Dimensions∘ Jordan, A., Krauss, S., Löwen, K., Blum, W., Neubrand, M., Brunner, M., Kunter, M., & Baumert, J. (2008). Aufgaben im COACTIV-Projekt: Zeugnisse des kognitiven Aktivierungspotentials im deutschen Mathematikunterricht

cognitive activation

in general teaching

in math teaching

in math tasks

in math task feedback

- from characteristics details, applicable for activating feedback design
  - no
    - small step or exact instructions for solving tasks
    - immediate information about right or wrong
  - but
    - open questions which stimulate contemplation
    - prompts to provide reasons for or to think about what results from answers
    - comparing and evaluating different task solutions

Jordan, A., Krauss, S., Löwen, K., Blum, W., Neubrand, M., Brunner, M., Kunter, M., & Baumert, J. (2008). Aufgaben im COACTIV-Projekt: Zeugnisse des kognitiven Aktivierungspotentials im deutschen Mathematikunterricht ∘ Leuders, T., & Holzäpfel, L. (2011). Kognitive Aktivierung im Mathematikunterricht

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time for
trying examples?

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PHHD Demo Tasks

Dashboard / Meine Kurse / demo / Allgemeines / demo tasks

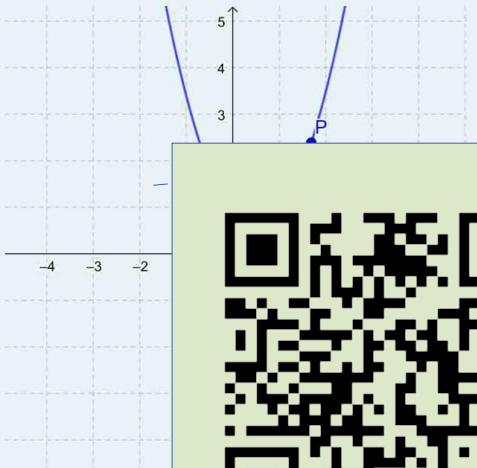
Frage **3**

Unvollständig

Erreichbare Punkte: 1,00

Frage markieren

Verschiebe die Punkte S und P so, dass der Graph zur Funktion $f(x) = -2 \cdot (x-3)^2 + 3$. passt.



Prüfen

Test-Navigation

1 2 3 4 5 6

Versuch abschließen ...

demo tasks here

stack.authomath.org

login: ws24stackXXX
PW: ws24stackXXX

kurs: demo

your feedback here

overview

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activating feedback

vom

- informing
  about (parts of)
  necessary knowledge

zum

- initiating
  self-directed (re)construction
  of knowledge

activating feedback

As you know,

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2$$
$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2$$
$$(a-b) \cdot (a+b) = a^2 - b^2$$

Now factorise $18 \cdot s^2 + 24 \cdot s \cdot t + 8 \cdot t^2$ by using one of the three formulas above.

You can do your calculations here:

```
18*s^2+24*s*t+8*t^2
= (18*s+8*t)^2
```

Denote your solution here:

```
(18*s+8*t)^2
```

Wrong, too bad.

Correct would be $2 \cdot (3 \cdot s + 2 \cdot t)^2$

That's how to do it:

Here is the expression again:

$18 \cdot s^2 + 24 \cdot s \cdot t + 8 \cdot t^2$

First, you need to find two square numbers.
You can identify them once you factor out 2:

$= 2 \cdot (9 \cdot s^2 + 12 \cdot s \cdot t + 4 \cdot t^2)$

Now the square numbers are visible inside the brackets: 9 und 4

Second, choose from the three formulas mentioned above the one that has the same structure as the expression inside the brackets:

$9 \cdot s^2 + 12 \cdot s \cdot t + 4 \cdot t^2$
corresponds to
$a^2 + 2 \cdot a \cdot b + b^2$

Third, identify the corresponding parts of each expression:

a^2 corresponds to $9 \cdot s^2$, hence $a = 3 \cdot s$, and
b^2 corresponds to $4 \cdot t^2$. So $b = 2 \cdot t$

And check whether $2 \cdot a \cdot b$ corresponds to $12 \cdot s \cdot t$:
$2 \cdot 3 \cdot s \cdot 2 \cdot t = 12 \cdot s \cdot t$,
which hence is the case.

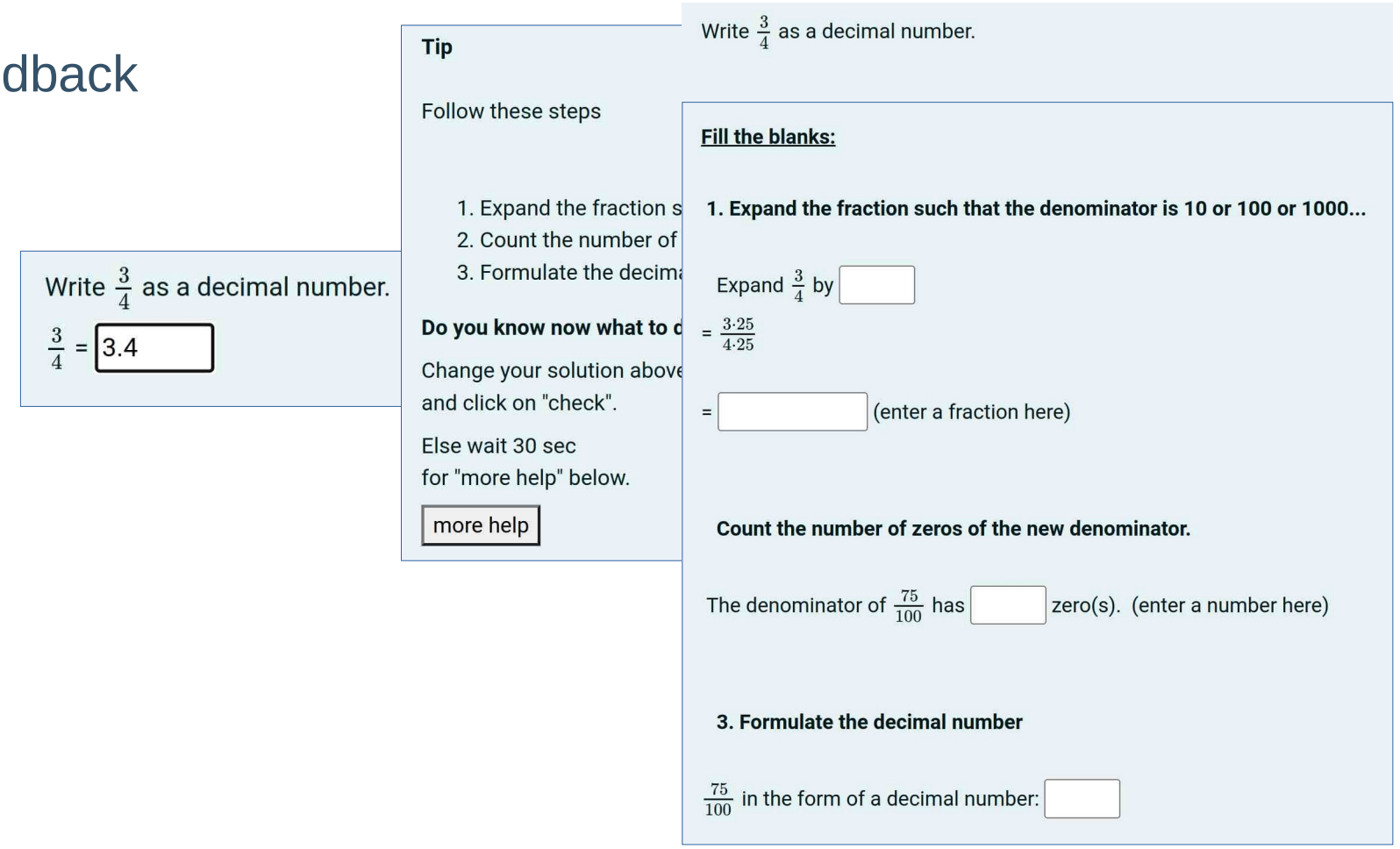
Fourth, substitute the values for a and b in $(a+b)^2$.
And do not forget the factor from the first step to denote the final solution:

$= 2 \cdot (3 \cdot s + 2 \cdot t)^2$

Try this task again!!

Hoch & Dreyfus (2005): Students' Difficulties with Applying a Familiar Formula in an Unfa tructural perspective.

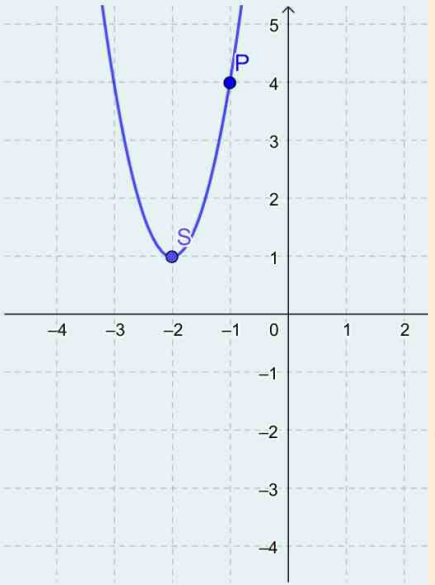
activating feedback



Gulden (2020): Entwicklung eines gestuften Feedbacks für digitale Bruchrechenaufgaben

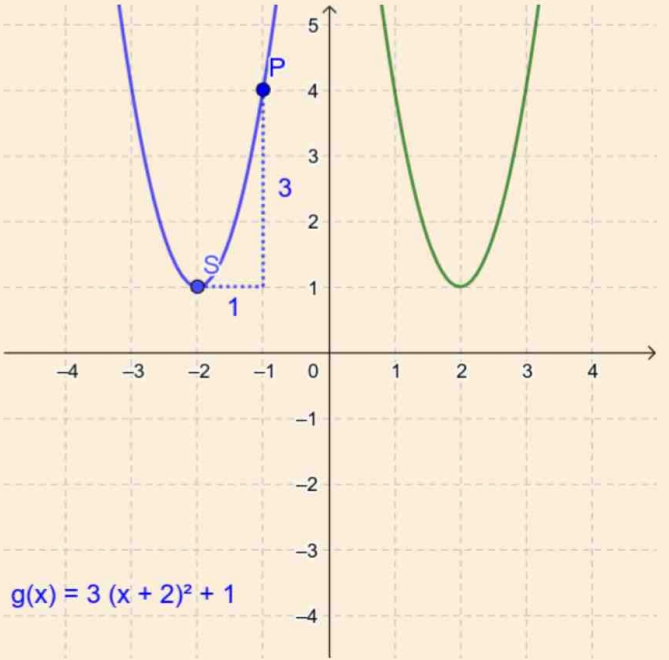
activating feedback

Verschiebe die Punkte S und P so,
dass der Graph zur Funktion
$f(x) = 3 \cdot (x-2)^2 + 1.$
passt.



Wrong, unfortunately.

The green graph would be correct:



Why?

Find out yourself:

Place your blue graph
onto the green graph
and follow closely
how the expression changes.

Does this help you?

Then try this task again.

Else wait for 30 seconds,
and some more help appears here:

further help

activating feedback

Give a quadratic expression
which has exactly the two roots -3 und -1 .

$f(x) =$ (x-3)*(x-1)

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer again...

Does this help you?

Then try this task again.

Else wait for 30 seconds,
and some more help appears here:

further help

activating feedback

- statements, propositions, description
- pictures, graphs
- videos, movies

- clozes, scaffolding

- questions, hints, food for thought
- interactive elements for exploration

> “Unless students see themselves as agents of their own change […] they may neither be receptive to useful information about their work, nor be able to use it.”
>
> Boud & Molloy (2013)

> “Interactive feedback is more effective than other kinds of feedback in improving students’ performance.”
>
> Barana, Marcisio & Sacchet (2021)

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feedback design

Addressing

Adapting

Activating

Structuring

feedback design

Addressing

Adapting

Activating

Structuring

addressing object of learning

from

- addressing procedures
  that are necessary to master
  the given task

to

- providing the conceptual basis
  for understanding the given
  and related tasks

addressing object of learning

Calculate:

$\frac{1}{2} + \frac{1}{5} =$ 2/10

Wrong, sorry!

You have found a common denominator.

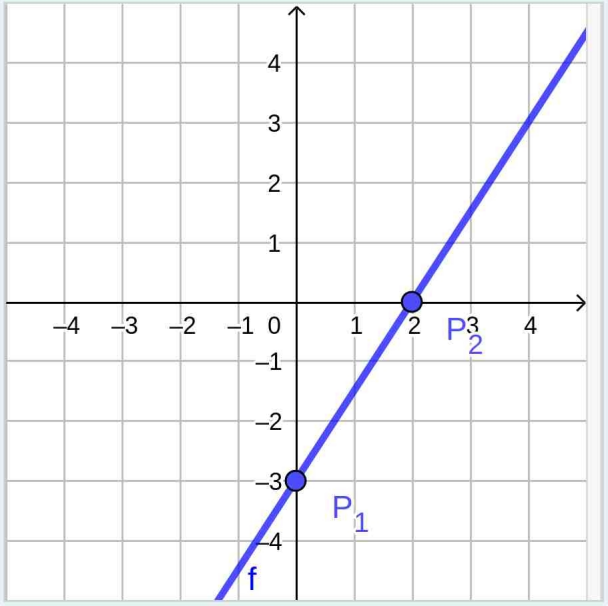
But also expand the numerators:

$$\frac{1}{2} + \frac{1}{5} = \frac{1 \cdot 5}{2 \cdot 5} + \frac{1 \cdot 2}{5 \cdot 2}$$

addressing object of learning

Give the graph to the function
$f(x) = 2 \cdot x - 3$.

Place P_1 and P_2
such that the line fits the expression.



Follow these steps:

1. Place P_1

The number -3 in $f(x) = 2 \cdot x - 3$
marks the place on the y-axis.

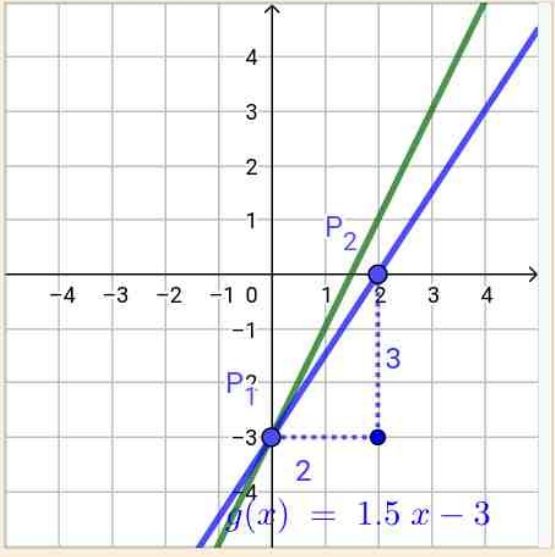
Place P_1 here.

2. Place P_2

The other number 2 in $2 \cdot x - 3$
denotes the slope of the line.

Hence start with P_2 in P_1,
then move P_2 one step to the right,
and after that move 2 steps vertically.

Place P_2 here.



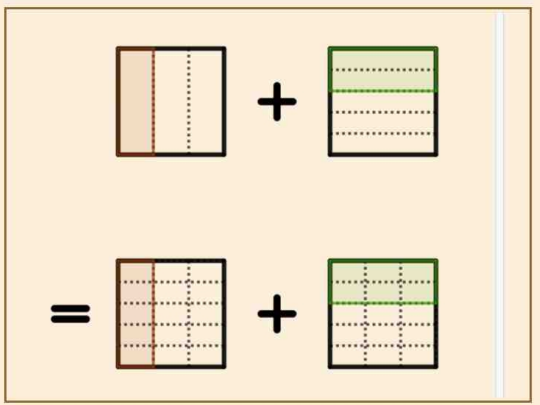
addressing object of learning

Calculate:

$\frac{1}{3} + \frac{2}{5} =$ 3/8

Wrong, sorry!

Maybe this visualisation
of the first step
helps you to find your mistake?



$\frac{1}{3} + \frac{2}{5}$

$= \frac{5}{15} + \frac{6}{15}$

addressing object of learning

Solve:

$2 \cdot (q + 1) = 4$

Copy the equation below,
then note each next step beneath:

2*(q+1)=4
2*q+2=4
2q=2
q=1

$L = \{$ 1 $\}$

Good. Your solution is correct.
And the transformations are fine.

But that took long!

There is a faster solution - compare:

$2 \cdot (q + 1) = 4$	$2 \cdot (q + 1) = 4$
$2 \cdot q + 2 = 4$	$q + 1 = 2$
...	...

One is your strategy,
the other is faster.

Well? Do you have an idea?

Then try this task again.

Else wait for 30 sec,
then a full solution appears:

Click here for full solution

Weigand, Schüler-Meyer & Pinkernell (2022): Didaktik der Algebra ∘ Wolff (2018): Umformen und Lösen von quadratischen Gleichungen ∘ Rüede (2013): How secondary level teachers and students impose personal structure on fractional expressions and equations ∘ Rittle-Johnson & Star (2009): Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving.

addressing object of learning

- worked out
  solving procedure
- specific reference
  to single steps
- interactive scaffolding
  through steps
- references
  to relevant rules

- explanatory models
  (“Grundvorstellungen”)
- representational
  or contextual flexibility
- strategic flexibility

addressing object of learning

- worked out solving procedure
- specific reference to single steps
- interactive scaffolding through steps

mastery of procedures reduce cognitive load while solving complex and challenging problems

"a deep understanding of learning involves the construction of meaning (understanding) and relates more to the relationships, cognitive processes, and transference to other more difficult or untried tasks" (Hattie & Timperley, 2007)

- explanatory models ("Grundvorstellungen")
- representational or contextual flexibility
- strategic flexibility

feedback design

Addressing

Adapting

Activating

Structuring

adaption to subject and object of learning

from

- the same feedback
  for each answer

to

- specific feedback
  for each answer case

adaption to subject and object of learning

Give a cubic expression
which has exactly the two roots 1 und 4 .

$f(x) =$ (x-4)*(x-1)

Wrong, too bad.

A correct expression would be $(x-4)^2 \cdot (x-1)$.

Why is that?

You need to know

that a linear expression like $(x-a)$ has a as root,
that $(x-a) \cdot (x-b)$ is a quadratic expression and has a und b as roots,
and that $(x-a) \cdot (x-b) \cdot (x-c)$ is a cubic expression with roots a, b und c.

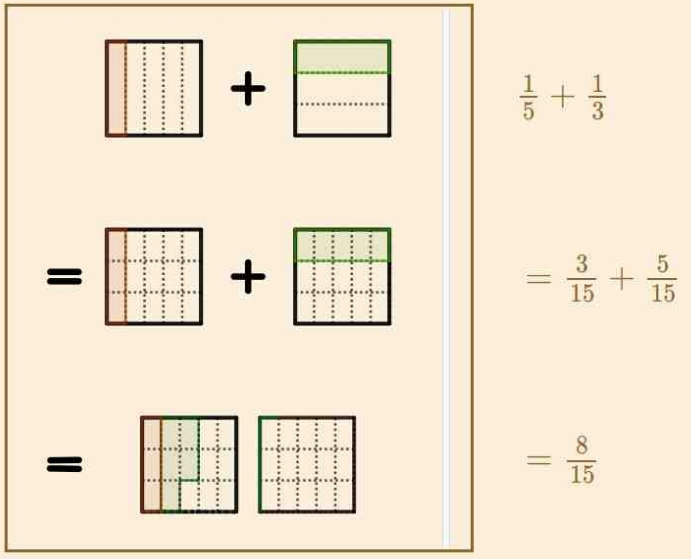
adaption to subject and object of learning

Calculate:

$\frac{1}{5} + \frac{1}{3} =$ 2/8

Wrong, I am afraid.

This visualization should help you to understand:



And also reduce the fraction,
if necessary.

adaption to subject and object of learning

Calculate:

$\frac{1}{2} + \frac{1}{5} =$ 2/10

Wrong, sorry!

You have found a common denominator.

But also expand the numerators:

$$\frac{1}{2} + \frac{1}{5} = \frac{1 \cdot 5}{2 \cdot 5} + \frac{1 \cdot 2}{5 \cdot 2}$$

adaption to subject and object of learning

Give a quadratic expression
which has exactly the two roots -3 und -1 .

$f(x) =$ (x-3)*(x-1)

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer again...

adaption to subject and object of learning

- basic
  declarative,
  procedural or
  conceptual knowledge
  for mastering
  all varieties of the task

- specific advice
  for a priori identified
  answer cases
- specific advice
  depending on
  task properties

feedback design

Addressing

Adapting

Activating

Structuring

models

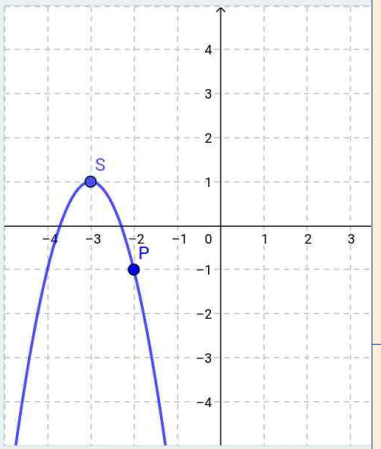
worked solution

Sorry, wrong (KR)
Correct would be… (KCR)
This is how to do it correctly: … (KH)

appears without delay

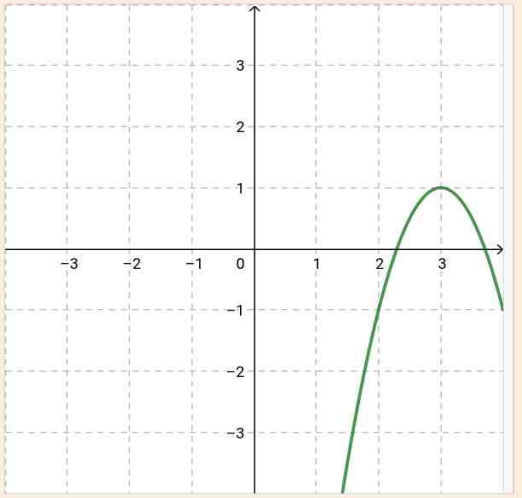
Try again? Click here:

Verändere die Position der Punkte S und P so, dass der Graph zur Funktion f mit $f(x) = -2 \cdot (x-3)^2 + 1$ passt.



Leider falsch.

Richtig wäre der grüne Graph.



So geht's:

Die Funktionsgleichung lautet ja $f(x) = -2 \cdot (x-3)^2 + 1$.

1. Platziere zuerst den Punkt S:

3 und 1 sind die Koordinaten des Scheitelpunktes.
Man findet sie im Term mit umgekehrten Vorzeichen in der Klammer und als zuletzt angegebene Zahl.

Platziere also S so,
dass er die Koordinaten 3 und 1 hat.

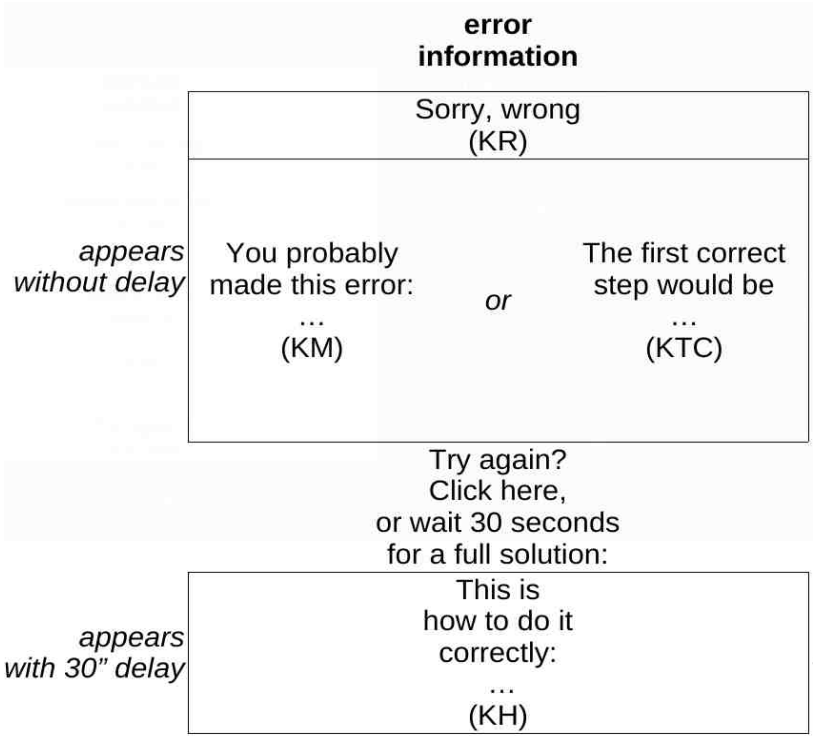
2. Platziere jetzt P:

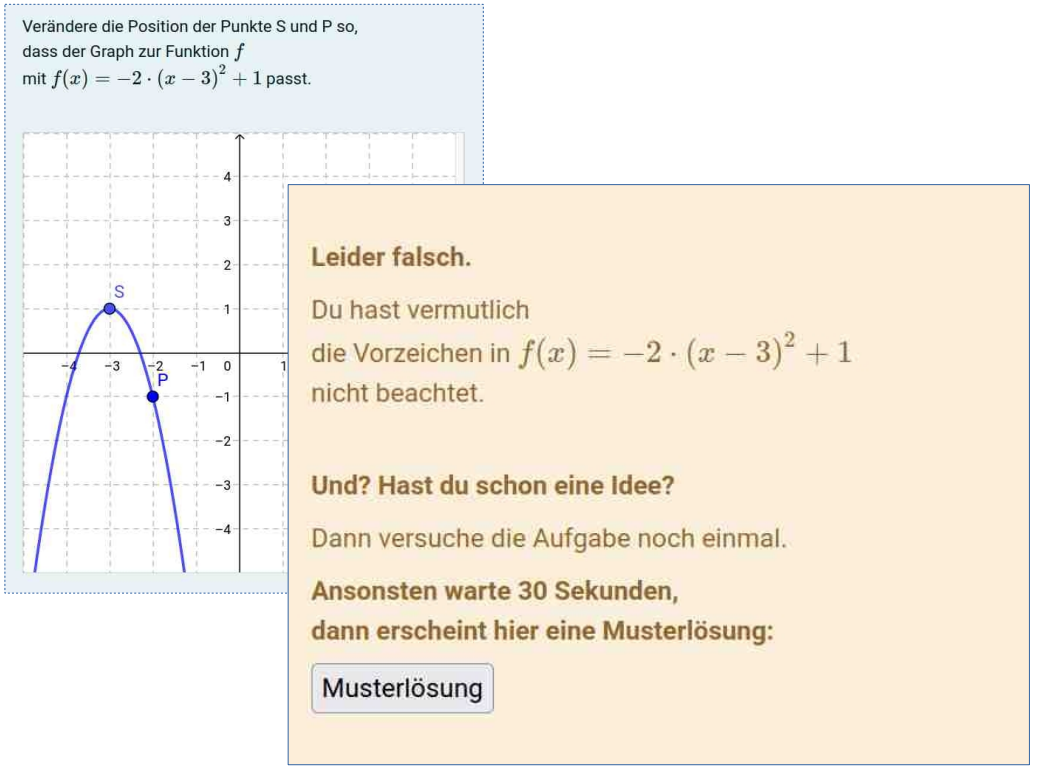
-2 steht für die Parabelöffnung.

Hierzu geht man von S einen Schritt nach rechts oder links und dann 2 Schritte nach unten.

Pinkernell, Gulden & Kalz (2020) : Automated feedback at task level: error analysis or worked out examples

models





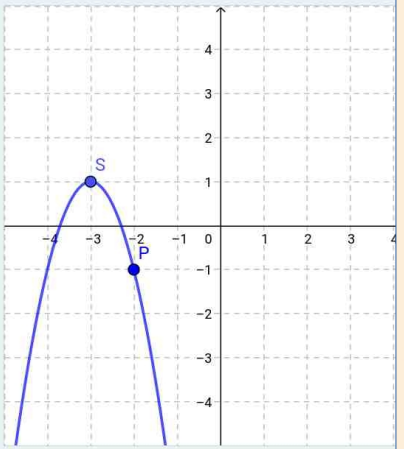
Pinkernell, Gulden & Kalz (2020) : Automated feedback at task level: error analysis or worked out examples

models

activating feedback

	Sorry, wrong (KR)
appears without delay	Find out yourself how to do it right: Try this activity *or* Think about this *or* Compare your answer with the correct solution
	Have an idea? Click here for trying again, or wait 30 seconds for a full solution:
appears with 30" delay	This is how to do it correctly: … (KH)

Verändere die Position der Punkte S und P so, dass der Graph zur Funktion f mit $f(x) = -2 \cdot (x-3)^2 + 1$ passt.



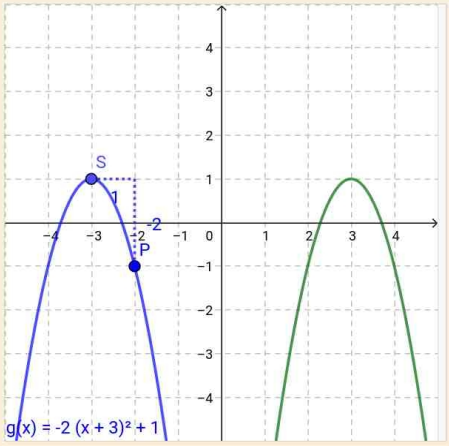
Leider falsch.

Richtig wäre der grüne Graph.

Warum?

Das kannst du selbst herausfinden

Korrigiere deinen blauen Graphen
und achte darauf, wie der Term sich ändert!



Beantworte dabei für dich die folgenden Fragen:

1. Wo im Term erkennt man die Koordinaten des Scheitelpunkts?

2. Wo im Term erkennt man die Öffnung der Parabel wieder?
   Die Öffnung ist übrigens die Länge der senkrechten Seite des gestrichelten Dreiecks, wenn die horizontale 1 lang ist.

Und? Hast du schon eine Idee?

Dann versuche die Aufgabe noch einmal.

Ansonsten warte 30 Sekunden, dann erscheint hier eine Musterlösung:

Musterlösung

models

feedback
referring to explanatory models

	Sorry, wrong (KR)
appears without delay	This helps you to understand the concept: [explanatory model]
	Have an idea? Click here for trying again, or wait 30 seconds for a full solution:
appears with 30" delay	This is how to do it correctly: … (KH)

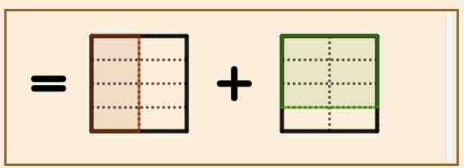
Calculate:

$\frac{1}{2} + \frac{3}{4} =$ 4/6

Too bad, not fully correct.

Why is that?

Maybe this translation of the second line gives you an idea?



$\frac{4}{8} + \frac{6}{8}$

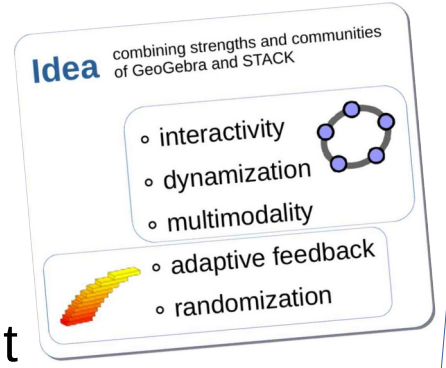
Do you know what to do now?

Then reload another question
and try again.

Or wait for 15 seconds for a full solution:

Click here for a full solution.

summary



1. what this is about
2. some background theory
3. activating feedback
4. feedback design parameter

- open questions which stimulate contemplation
- prompts to provide reasons for or to think about what results from answers
- comparing and evaluating different task solutions

Give a quadratic expression which has exactly the two roots -3 und -1.

$f(x) =$ (x-3)*(x-1)

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer again...

Addressing
Adapting
Activating
Structuring

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