Technical Advances Initiating Didactic Reflection in Teacher Education

∘ Workshop ∘

Guido Pinkernell ∘ Pädagogische Hochschule Heidelberg
ICTMT Athens ∘ June 2023

1. **AuthOMath : idea and objectives**

   – information I and activity I –

2. **Digital Math Task Design**

   – information II and activity II

3. **AuthOMath's DiCo : discussion**

1. **AuthOMath : idea and objectives**

   – information I and activity I –

2. **Digital Math Task Design**

   – information II and activity II

3. **AuthOMath's DiCo : discussion**

AuthOMath

Authoring Online Material with
Multimodal, Dynamic
and Interactive Applets
and Automated Feedback
for Learning Math

www.authomath.org
mathebruecke.pinkernell.online

AuthOMath

University of Education Heidelberg : Guido Pinkernell
University of Cantabria Santander : Jose Manuel Diego Mantecon
University of Edinburgh : Chris Sangwin
Johann-Kepler-Universität Linz : Zsolt Lavica
Geogebra GmbH (associated)

AuthOMath

AuTo

- a moodle based authoring tool
  for randomized interactive and dynamic
  multimodal mathematical tasks
  with automatic adaptive feedback

  which basically means to

  extend STACK to facilitate
  the implementation of
  GeoGebra applets into
  task and feedback

DiCo

- a didactical concept
  for designing online based
  interactive learning material
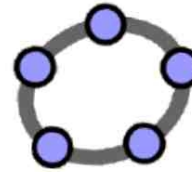 for use in mathematics teacher
  education

AuthOMath

AuTo

- a moodle base... ...oncept
  for randomized... ...online based
  multimodal ma... ...arning material
  with automatic... ...athematics teacher

  which basicall...

  extend STAC...
  the implemen...
  GeoGebra ap...
  task and feedback

Idea combining strengths and communities of GeoGebra and STACK

- interactivity
- dynamization
- multimodality
- adaptive feedback
- randomization

activity I

aim
work
give feedback

- gain insight into the type of
  digital math tasks AuthOMath is about

  by trying a collection of tasks from AuthOMath

- explore the field of possible criteria
  that need to be considered for
  "good digital math task design (in AuthOMath)"

  and share your thoughts

activity I

aim
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  digital math tasks AuthOMath is about

  by trying a collection of tasks from AuthOMath

- explore the field of possible criteria
  that need to be considered for
  "good digital math task design (in AuthOMath)"

  and share your thoughts here:

stack.authomath.org
> login : ictmt2023-XX
> passwd: same
> dashboard
> ICTMT

flinga.fi/s/FA5YLTZ

1. **AuthOMath : idea and objectives**

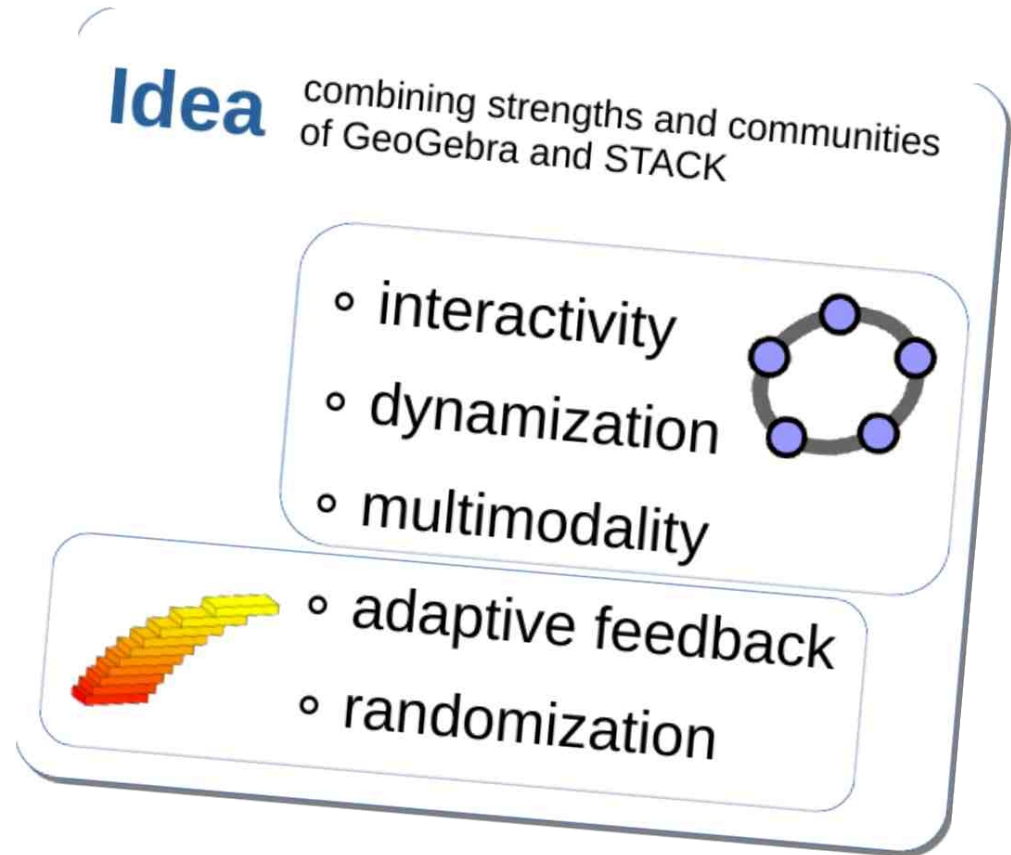   – information I and activity I –

2. **Digital Math Task Design**

   – information II and activity II

3. **AuthOMath's DiCo : discussion**

digital math task design

...in teacher education:

technical advances
resulting from combining
GeoGebra and STACK
initiate
didactic reflection

Idea combining strengths and communities of GeoGebra and STACK

- interactivity
- dynamization
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digital math task design

...in teacher education:

technical advances
resulting from combining
GeoGebra and STACK
initiate
didactic reflection

"You want to use
GeoGebra and STACK
for creating
multimodal and interactive
math tasks
with adaptive feedback.

What is it
that you need to think about
before you do the programming
(if this is you...)"

digital math task design

...in teacher education

1. how to address
   the learning object

2. how to address
   learners

3. how to use
   media

"You want to use
GeoGebra and STACK
for creating
multimodal and interactive
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What is it
that you need to think about
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(if this is you...)"

digital math task design

1. **how to address the learning object**

2. how to address learners

3. how to use media

- mathematical analysis:

  "do research on
  how to address the object of learning
  such that it supports
  mastery and understanding"

  focus on:

  - relevant definitions and terminology

  - mathematically valid explanatory models (aka basic ideas, "Grundvorstellungen")

  - specific representations, strategies, and applications

Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ◦ vom Hofe & Blum (2016): "Grundvorstellungen" as a Category of Subject-Matter Didactics

digital math task design

1. **how to address the learning object**

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The *WiGORA* framework

Declarative Knowledge — ability to recall rules, definitions and characteristic properties

Explanatory Models — ability to recall explanatory models that make sense of a mathematical concept

Operational Flexibility — ability to apply, adapt and modify mathematical procedures for situational needs

Representational Flexibility — ability to switch within and between representational forms

Knowledge Application — ability to identify a concept or procedure as suitable for a given problem

Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ vom Hofe & Blum (2016): "Grundvorstellungen" as a Category of Subject-Matter Didactics

digital math task design

1. **how to address the learning object**

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Move the points S und P, such that the graph fits with
$$f(x) = -3 \cdot (x+1)^2 - 2.$$

This picture explains how two fractions are added.

Translate into maths:
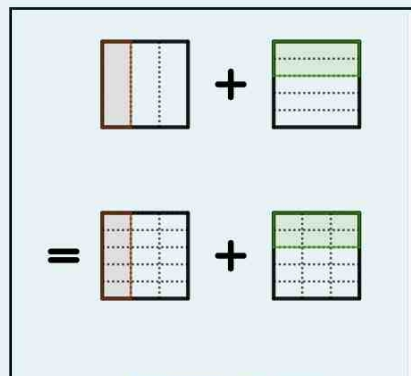
+

□ + □

= +

= □ + □

= □

Give a quadratic expression which has exactly the two roots -3 und -1.

$$f(x) = \boxed{(x-3)*(x-1)}$$

"Multiply the difference of 2 and x with 4 and you get 8."

Translate into an equation:

$\boxed{2\text{-}x*4 = 8}$

Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ vom Hofe & Blum (2016): "Grundvorstellungen" as a Category of Subject-Matter Didactics
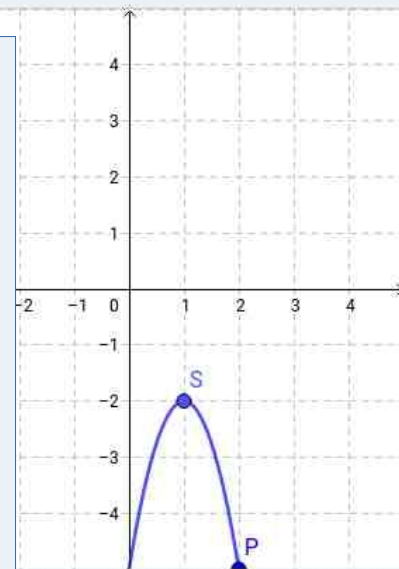
digital math task design

1. how to address
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- didactic perspective:

  "do research on
  how learners actually do access
  the object of learning
  (correctly or wrongly)"

  focus on

  – as novices or experts in the topic,
    as low or high achievers in general

  – individual conceptions,
    misconceptions,
    systematic errors

  – range of
    possible solving strategies

Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ Prediger (2008): Discontinuities for mental models - A source for difficulties with the multiplication of fractions ∘ Rittle-Johnson & Star (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving

digital math task design

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Give a quadratic expression
which has exactly the two roots -3 und -1 .

$f(x) =$ | (x-3)*(x-1) |

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer again...

Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ Prediger (2008): Discontinuities for mental models - A source for difficulties with the multiplication of fractions ∘ Rittle-Johnson & Star (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving

digital math task design

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"Multiply the difference of 2 and x with 4 and you get 8."

Translate into an equation:

`2-x*4 = 8`

Well, yes and no.

You have translated all the words correctly into algebra.

But you should think of
"the difference of 2 and x" as a whole
that needs to be multiplied with 4.

Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ Prediger (2008): Discontinuities for mental models - A source for difficulties with the multiplication of fractions ∘ Rittle-Johnson & Star (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving

digital math task design

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Solve:

$$2\cdot(q+1)=4$$

Copy the equation below,
then note each next step beneath:

```
2*(q+1)=4
2*q+2=4
2q=2
q=1
```

$$2\cdot(q+1)=4$$
$$\Leftrightarrow \quad 2\cdot q+2=4$$
$$\Leftrightarrow$$
$$\Leftrightarrow$$

$$L=\{\,1\,\}$$

Good. Your solution is correct.
And the transformations are fine.

But that took long!

There is a faster solution - compare:

$2\cdot(q+1)=4$	$2\cdot(q+1)=4$
$2\cdot q+2=4$	$q+1=2$
...	...

One is your strategy,
the other is faster.

Well? Do you have an idea?

Then try this task again.

Else wait for 30 sec,
then a full solution appears:

Pinkernell (2019): Conceptualising knowledge of mathematical concepts or procedures ∘ Prediger
source for difficulties with the multiplication of fractions ∘ Rittle-Johnson & Star (2009). Compared
conceptual knowledge and procedural flexibility for equation solving

digital math task design

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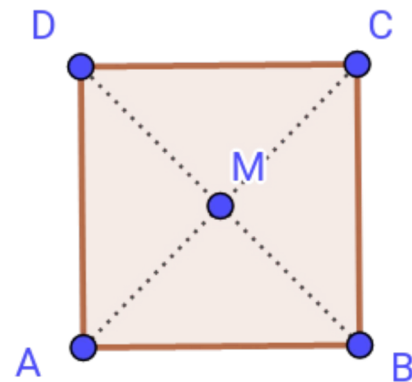
3. **how to use media**

- media perspective:

  "now decide on the use and structure of textual and pictorial elements of task and feedback area"

  focus on

  – how to use language

  – how to use static, dynamic, interactive elements

  – how to structure task and feedback

Yerushalmy (2005): Functions of Interactive Visual Representations ∘ Duval (1999): Representation, Vision and Visualization ∘ Pinkernell, G., Diego Mantecón, J. M., Lavicza, Z., Sangwin, C. (2023): AuthOMath: Combining the strengths of STACK and GeoGebra

digital math task design

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This is not a square.

Move points
to explore the range of appearances,
and then decide
what this quadrangle really is.

Yerushalmy (2005): Functions of Interactive Visual Representations ∘ Duval (1999): Representation, Vision and Visualization ∘
Pinkernell, G., Diego Mantecón, J. M., Lavicza, Z., Sangwin, C. (2023): AuthOMath: Combining the strengths of STACK and GeoGebra

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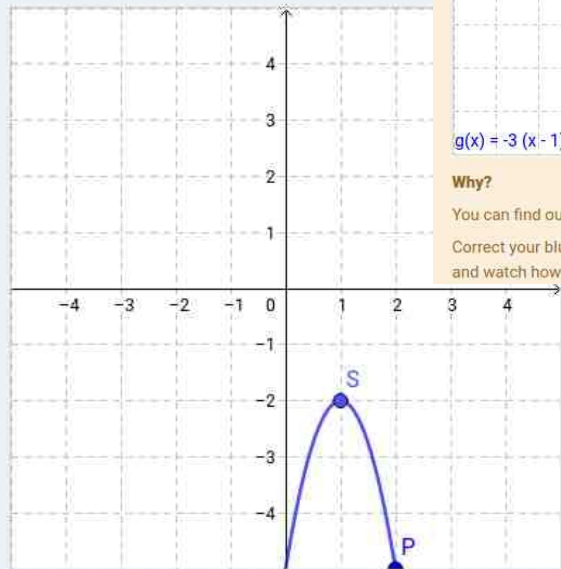
Check

Wrong, too bad!

The green graph would be correct.

g(x) = -3 (x - 1)² -2
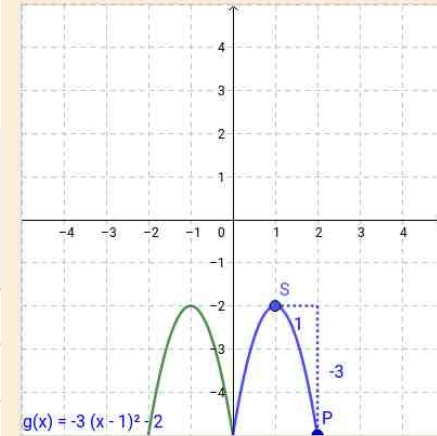
Why?

You can find out yourself.

Correct your blue graph and watch how the expression changes

Try to find answers to the following questions:

1. Where in the expression can you see the coordinates of the vertex?

2. Where in the expression can you see a value for the opening of the parabola?

Do you have an idea already?

Then try the task again.

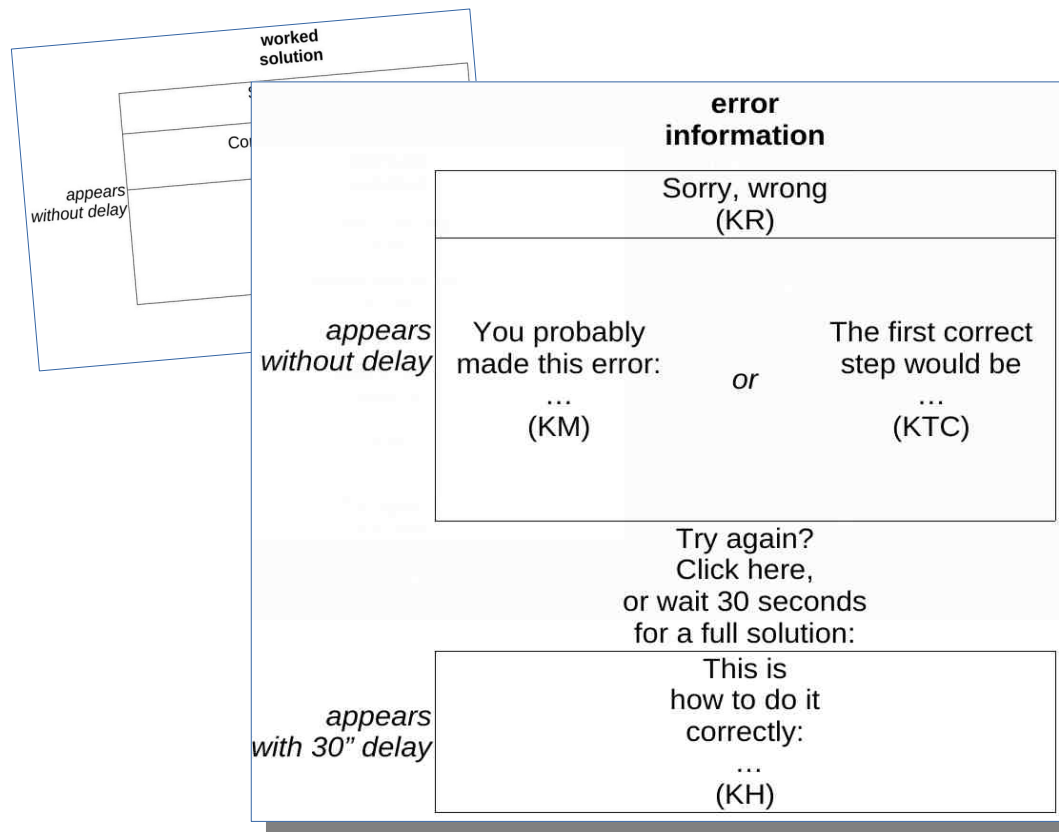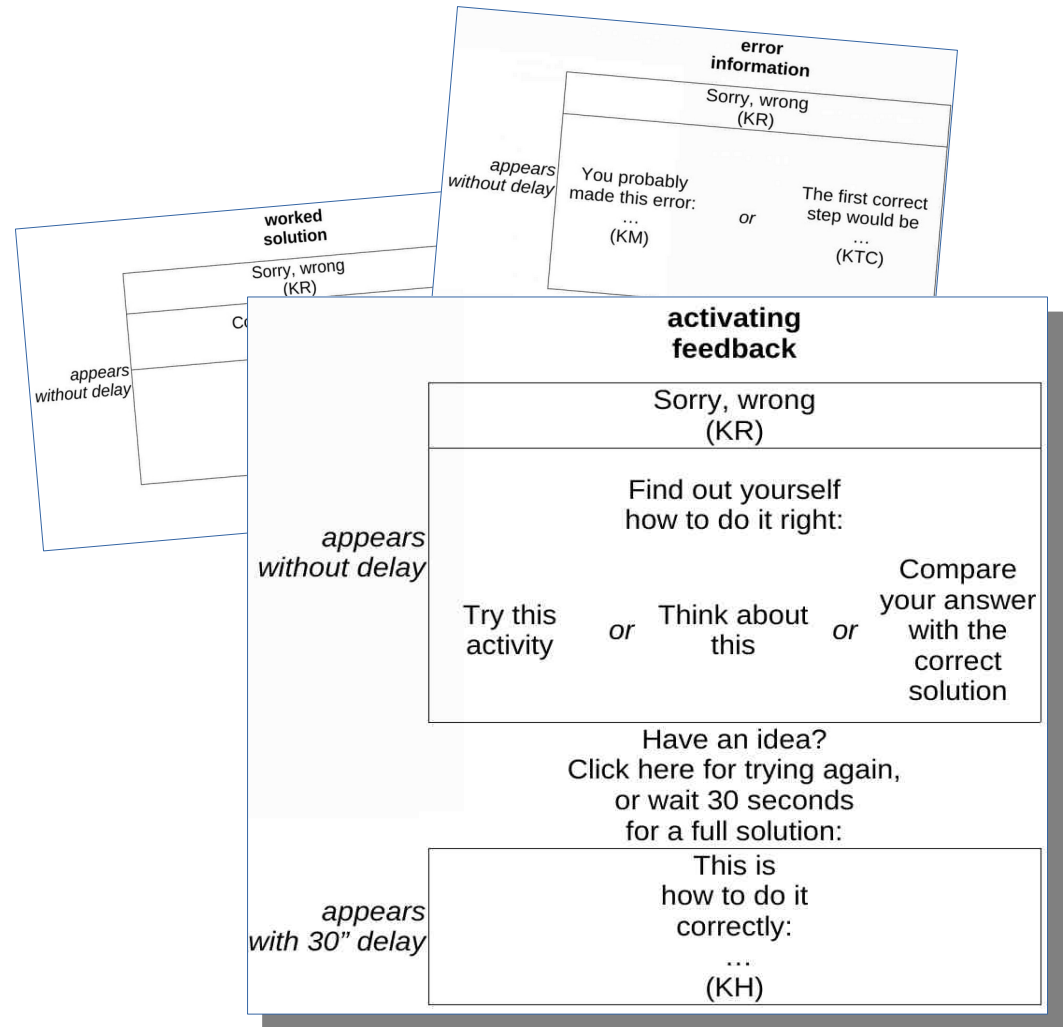**Or wait 30 seconds** after which a full solution will appear:

Musterlösung

digital math task design

1. how to address
   the learning object

2. how to address
   learners

3. **how to use
   media**

worked
solution

| Sorry, wrong (KR) |
| Correct would be… (KCR) |
| This is how to do it correctly: … (KH) |

appears without delay

Try again?
Click here:

digital math task design

1. how to address the learning object
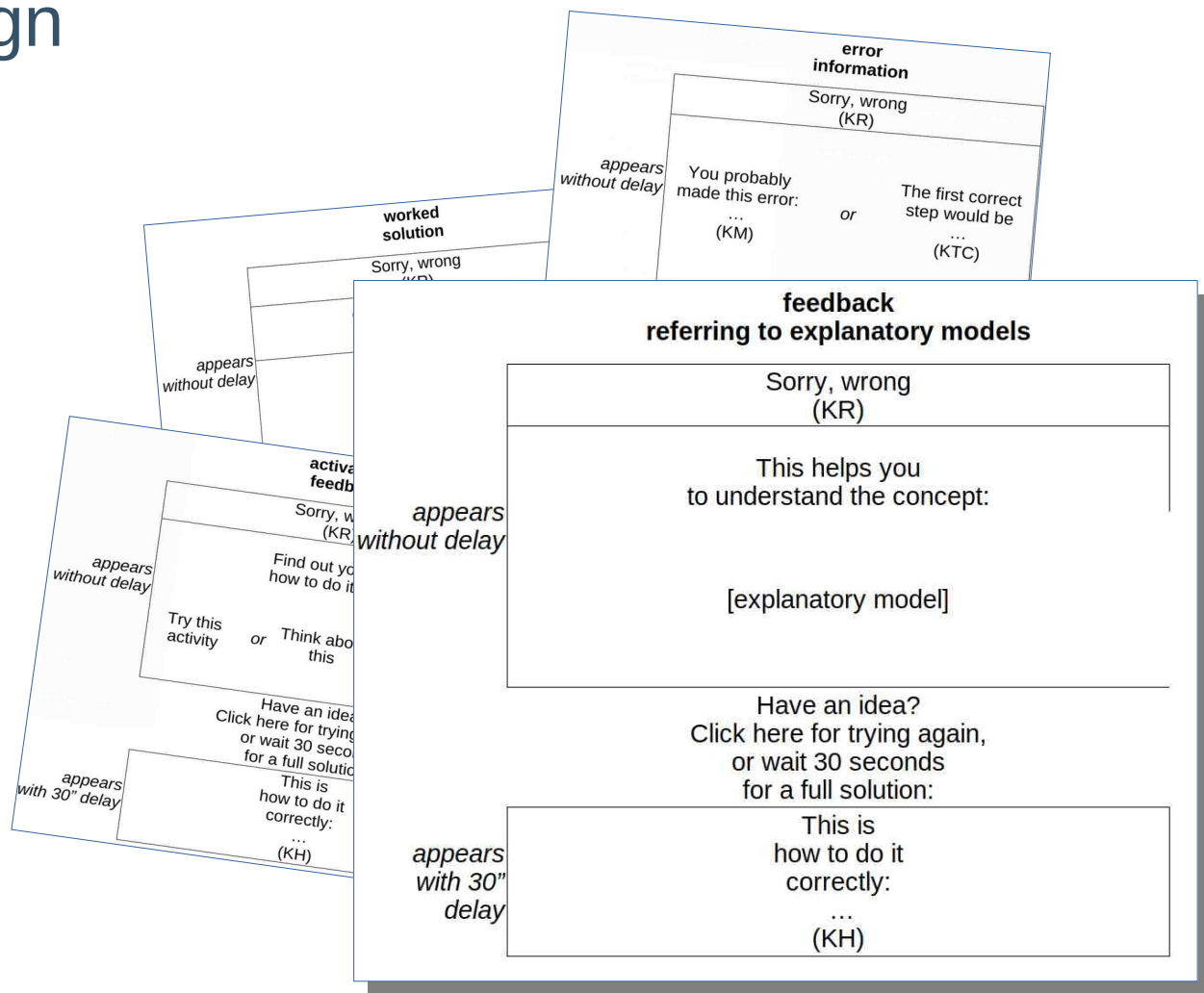
2. how to address learners

3. **how to use media**

worked
solution

error
information

Sorry, wrong
(KR)

appears without delay

You probably
made this error:
…
(KM)

or

The first correct
step would be
…
(KTC)

Try again?
Click here,
or wait 30 seconds
for a full solution:

appears with 30" delay

This is
how to do it
correctly:
…
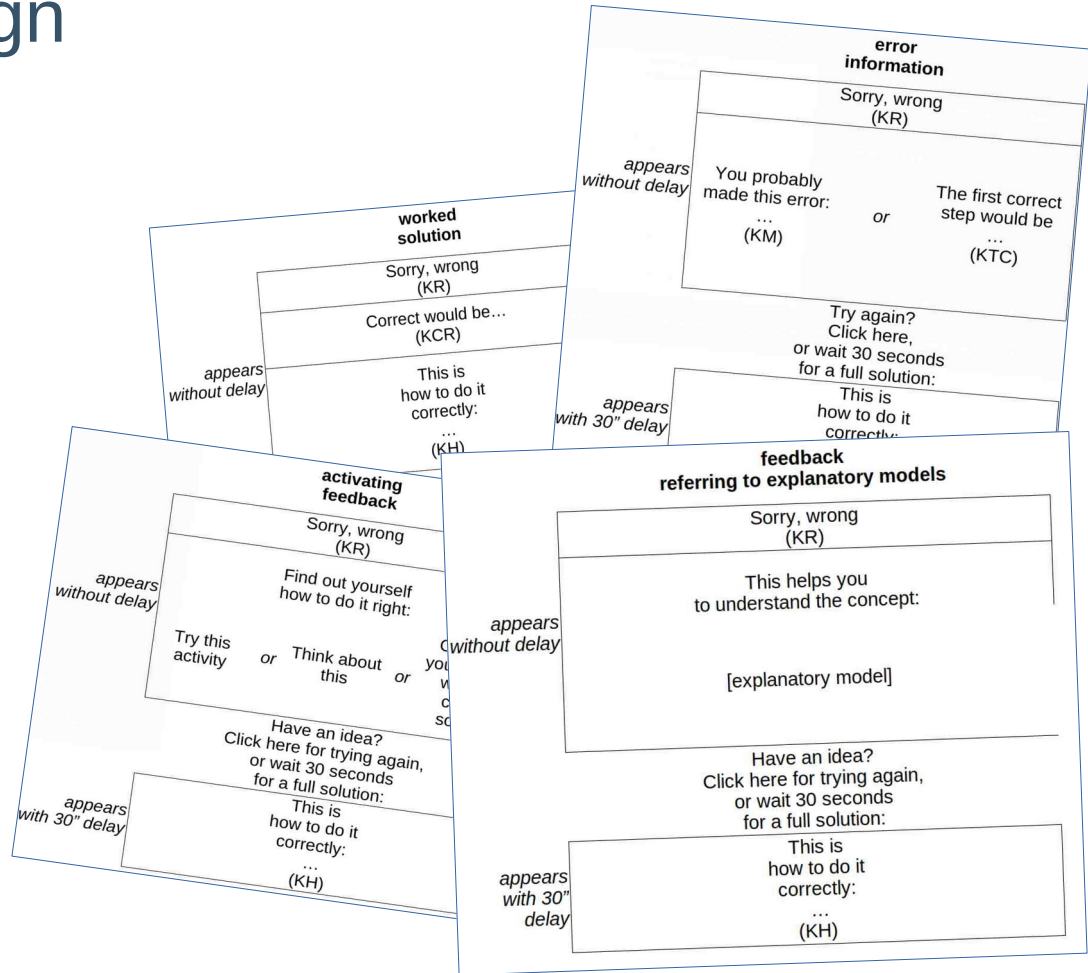(KH)

appears without delay

Co

digital math task design

1. how to address the learning object

2. how to address learners

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digital math task design

1. how to address the learning object

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activity II

aim
work
present

- see whether the three given aspects of digital math task design help with designing AuthOMath tasks

  by actually designing an AuthOMath task

  choose from these topics

  1 fractions, roots, potencies
  2 expressions and equations
  3 functions
  4 measuring lengths, areas, volumes
  5 geometric objects and concepts
  6 data and probability

  and go to your topic here:

https://docs.google.com/
document/d/
1gqJSegvvRWC_fYlGyg6owiZqVBF
WtPt8/edit?
usp=sharing&ouid=105668954735
657769599&rtpof=true&sd=true

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