



Formative feedback on linear equations

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Literary review

**Formative
feedback**

STACK

**Linear
equations**

Literary review: formative feedback



Feedback

“Information [...] regarding aspects of one’s performance or understanding”



Types

- Summative/**formative**
- Conceptual/**procedural** /based on practices
- Immediate**/delayed...



Given...

...after initial instruction.

“Feedback has no effect in a vacuum”.

It has to be useful.

Literary review: STACK



Answer: algebraic expressions



Encoding of typical mistakes



Specific feedback

$$\begin{aligned} & 3(-2x + 1) = -x \\ ? & -6x + 1 = -x \\ \Leftrightarrow & 1 = 5x \\ \Leftrightarrow & x = \frac{1}{5} \end{aligned}$$

Check whether you correctly applied the distributive property. Did you multiply the factor outside of the parentheses by each of the ones inside?

Literary review: linear equations

Distributive property

Expand $A(Bx+C)$

Addition

Add/subtract the same term on both sides

Multiplication

Multiply/divide by the same factor on both sides

Common denominator

Add fractional numbers

Coeff-var separation

$Ax \rightarrow A+x$

Arithmetic

Basic arithmetic operations

Pérez et al. (2019)



Main goals

To what extent is formative feedback through STACK effective for students overcoming difficulties on linear equations?



Methodology

Sample

132 students:

- 39 in secondary school (aged 14-16)
- 93 who have finished secondary school

Average achievers – some especially struggled

 Cantabria, Spain



Methodology

Equations

a) $3(-2x+1)=-x$

b) $\frac{7}{2}x = 14$

c) $-4x=16$

d) $-2(3x-4)=10$

e) $\frac{x}{3} + 2x = 7$

f) $-3x+5=17$



Pérez et al. (2019)

Feedback

$$3(-2x+1) = -x$$

? $-6x+1 = -x$

$\Leftrightarrow 1 = 5x$

$\Leftrightarrow x = \frac{1}{5}$

Check whether you correctly applied the distributive property. Did you multiply the factor outside of the parentheses by each of the ones inside?

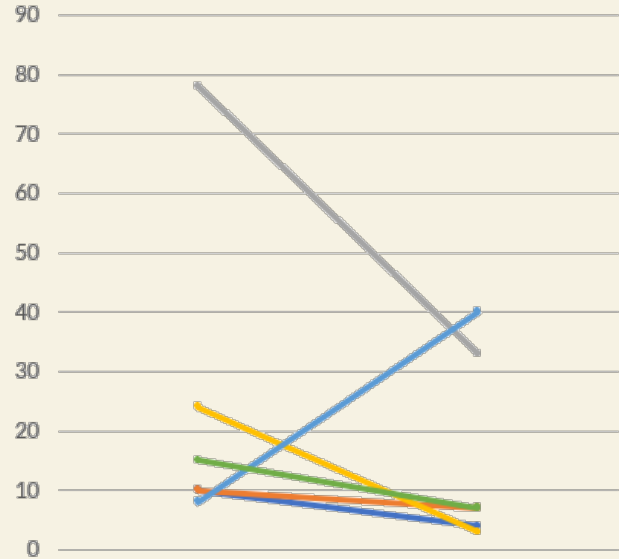


Results

Mistakes

- Distributive property ●
- Addition ●
- Multiplication ●
- Common denominator ●
- Coeff-var separation ●
- Arithmetic ●

Number of mistakes



Start

Finish

Results

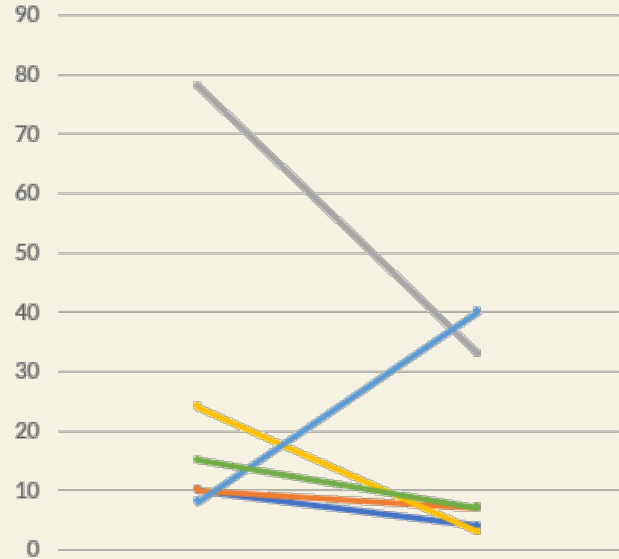
Distributive
property
Coeff-var
separation

Older students who
have forgotten

Addition
Arithmetic

Distractions

Number of mistakes



Start

Finish

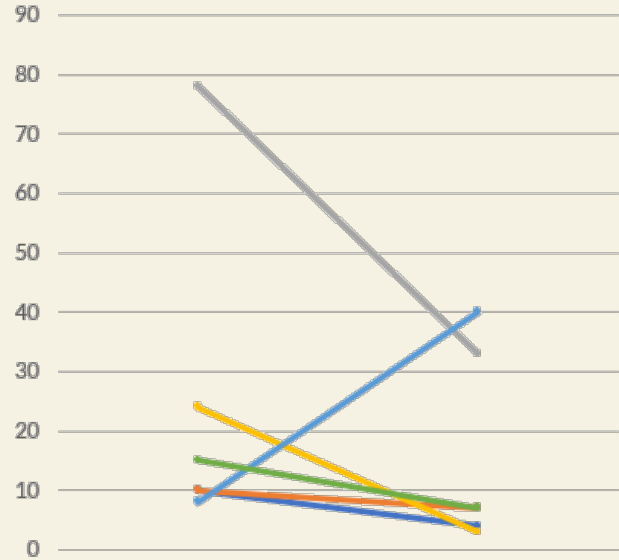
Results

Multiplication

Students tried to solve $Ax=B$ as:

- $x=B/A$
- $x=-B/A$
- $x=A/B$
- $x=-A/B$

Number of mistakes



Start

Finish

Results

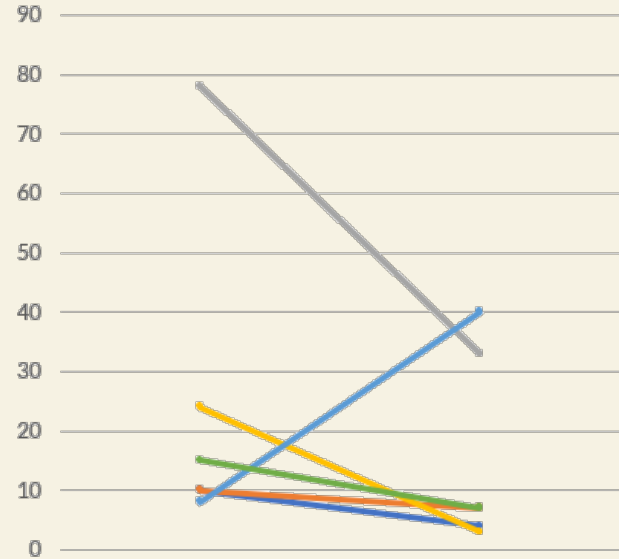
Common denominator

Students avoided using common denominator:

$$\begin{aligned} \frac{7}{2}x &= 14 \\ \Leftrightarrow 7x &= 28 \\ \Leftrightarrow x &= \frac{28}{7} \\ \Leftrightarrow x &= 4 \end{aligned}$$

$$\begin{aligned} \frac{x}{3} + 2x &= 7 \\ \Leftrightarrow \frac{x}{3} &= 7 - 2x \\ \Leftrightarrow x &= 3(7 - 2x) \\ \Leftrightarrow x &= 21 - 6x \\ \Leftrightarrow 7x &= 21 \\ \Leftrightarrow x &= 3 \end{aligned}$$

Number of mistakes



Start

Finish



Conclusion

To what extent is formative feedback through STACK effective for students overcoming difficulties on linear equations?



Conclusion

Mistakes

Fewer and easier to correct

Why?



Conclusion

Mistakes

Fewer and easier to correct

Feedback

- Immediate
- Only after the mistakes
 - Focused on the task
- Short, clear and specific

Computer → objective

Try again

Not providing grades

Prior
knowledge

STACK

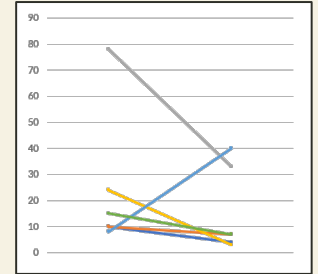
Conclusion

Mistakes

Fewer and easier to correct

Further studies

Different sample/feedback





References

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